Online Shopping Intermediaries:

The Strategic Design of Search Environments

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Abstract

An online shopping intermediary is an internet platform for consumers and third-party sellers to transact. Shopping intermediaries provide a search environment (e.g. search aids) to lower consumers' search costs in finding and evaluating sellers' products. In a theoretical model, we study strategic incentives of an intermediary in the design of its search environment as a means to ease search costs. An important aspect of our analysis is that consumers optimally decide how many sellers to evaluate and how deeply (e.g. number of attributes) to evaluate each of them. We find that the equilibrium search environment embeds enough search costs to prevent consumers from evaluating too many sellers, but not too much to prevent them from evaluating sellers' products partially. When facing consumers of heterogeneous search abilities, the search environment has all consumers evaluating products at full depth and consumers with higher evaluation abilities evaluating more sellers. We also show that intermediaries embed weakly less search costs with competition.

1. Introduction

Consumers increasingly use the internet to evaluate product information and make purchases. Following this trend, online intermediaries, such as Taobao Mall, YahooShopping and Amazon, offer platforms for consumers and third-party sellers to interact. And, to help consumers navigate and evaluate the huge number of sellers' products, they provide interactive search tools in their search environments. According to a manager from one of the above online platforms, serving both third-party sellers and consumers of these sellers creates two goals for any online intermediary: help consumers to find what they want easily, and keep sellers' competition in check. The problem, however, is that often times these are conflicting tasks and this raises the following managerial question: How should the intermediary design its search environment to help consumers find a desirable product while ensuring the profitability of their hosted sellers?

Unlike conventional retailers, online shopping intermediaries host third-party sellers, give them freedom to choose sale prices, and then typically charge a percentage of the final price as a referral fee. This revenue sharing scheme between intermediaries and sellers incentivizes intermediaries to protect sellers from fierce competition in order to benefit from higher prices. This research studies the strategic considerations of an online shopping intermediary in designing its search environment to balance the needs of its consumers and the benefits of its sellers. We focus, in particular, on the strategic aspects of search environment design, which is operationalized as a means to ease search costs,[1] as it affects consumers' evaluation incentives.

The conventional wisdom in the search literature suggests that consumers scale back the number of sellers they consider when faced with higher search costs and this softens sellers' price competition (c.f. Anderson & Renault 1999).[2] However, in reality, consumers always have the flexibility to decide the amount of information they acquire about any given product. The recent search literature suggests that consumers can react to high search costs by endogenously deciding how much to invest in information

[1] We use search costs and evaluation costs interchangeably.
[2] In the context of online shopping, Lal and Sarvary (1999) shows that the presence of the internet can increase the costs for consumers to evaluate another product at a physical store implying softened price competition.

acquisition of any given product (Branco, Sun and Villas-Boas 2012). Incorporating this within-product search dimension (or *evaluation depth* in the following text) makes consumers' evaluation incentives not transparent. In this paper, we study how the two-dimensional search – breadth (across sellers) and depth (within products) – is affected by the design of the search environment and what an intermediary's strategic considerations are in this design.

To illustrate the tradeoff of a consumer in her depth and breadth choice, consider a consumer who wants to buy a camera. She knows her tastes about cameras (e.g. style, color, size, viewfinder type, …) and her price range, but does not have a particular camera in mind. At the intermediary's website, she will evaluate a set of models offered by the third-party sellers. Since evaluating cameras is costly, it is up to the consumer to decide how many sellers to evaluate (*evaluation breadth*) and how much information to acquire for each model (*evaluation depth*). She may decide, for instance, to look into every technical detail available (e.g. megapixels, screen size, memory, aspect ratio etc.). But doing so, she might not be able to consider many camera brands. Alternatively, she may consider many brands but she might have to forego some technical details. A shopping intermediary (e.g. YahooShopping) can create an interactive search environment that affects this trade-off.

The depth and breadth of the consumer's *evaluation plan* affects her knowledge of the evaluated products. Specifically, evaluation depth affects how much uncertainty she has about her most preferred product and evaluation breadth affects how many sellers are competing for her purchase. In classic search models, a larger search cost implies that a consumer will evaluate fewer sellers (lower evaluation breadth). In our setting, however, a consumer can react to higher search costs by instead decreasing the depth of evaluation rather than by only cutting back on the number of evaluated sellers. Because of the endogenous interplay between evaluation depth and breadth, we can model the intermediary's strategic design of the search environment by its choice of conventional search costs.

Our model generates several new insights. First, we provide a necessary and sufficient condition for the intermediary's profit maximizing search environment. Specifically, the intermediary's optimal search environment maximizes consumer search cost subject to the condition that the consumer does not partially evaluate products. If search costs exceed this point, the consumer will evaluate products partially (e.g. not evaluating some product attributes), being unable to fully appreciate the value of her most preferred product. This puts downward pressure on sellers' prices. But, by making search less costly, the consumer broadens the set of sellers evaluated, inducing sellers to price more competitively. The intermediary's optimal search environment reflects, therefore, a balance between lowering search costs sufficiently so that the consumer fully evaluates the products she considers and fully knows what she is buying, but not too much that she searches a lot of sellers.

Second, we demonstrate that deepening evaluation depth plays a larger role in sustaining higher prices than limiting evaluation breadth. This is because deeper evaluation depth allows consumers to better appreciate the value of their most preferred product, which plays a direct role in increasing sellers' prices. In contrast, limiting evaluation breadth plays a weaker role in sustaining prices because it acts indirectly through sellers' competitive interactions. This result reinforces the importance of full evaluation in the optimality condition discussed above. This finding provides some guidance for the intermediary facing consumers with heterogeneous evaluation abilities. It suggests, in particular, that the major consideration of intermediary's design objective of search environment should be to err on the side of sufficiently lower search costs such that the consumers with lower evaluation abilities can evaluate products at full depth, although doing so allows the consumers with higher evaluation abilities to broaden their evaluation breadth.

The third new insight concerns the impact of product differentiation on the intermediary's optimal design. Less differentiation implies a lower benefit of product evaluation, to which consumers tend to react by scaling back evaluation depth – evaluating products only partially. To ensure full evaluation, the intermediary should provide a more helpful search environment to reduce search costs. However, both the intermediary and sellers are hurt by the reduced prices stemming from lower product differentiation.

Our final insight regards how competition among intermediaries affects the design of search environment. We find that the optimal search environment for a competitive intermediary is identical to the monopolist's, but only when intermediaries are relatively differentiated. If intermediaries are less differentiated, intermediaries provide a perfectly helpful search environment, with very low search costs. In this case, intermediaries are drawn into a prisoner's dilemma. That is, while they would jointly be better off providing more costly search environments, the intense competition means that each can acquire a substantial increase in consumer traffic at their sites by making their search environment more helpful.

Our work builds off the literature about accuracy and effort tradeoffs of a decision maker's decision rule selection. Specifically, people select a particular decision rule in a specific environment by weighing the costs and benefits of a set of rules, and adapt their decision making strategies to the specific environment (Payne 1982). Using this framework, Häubl & Trifts (2000) showed that the online shopping environment lowers consumers' efforts, allowing them to evaluate selected products at a greater depth and making better purchase decisions. However, their study focuses on how the search environment helps consumers' product evaluations, without regard to the pricing incentives of third-party sellers. In contrast, we propose an equilibrium model with a representative consumer, intermediaries, and competing sellers and study their strategic interactions in order to identify the intermediary's optimal level of search costs in its shopping environment.

Our methodology is based on the literature of partial product evaluation that has been growing in recent theory research (Bar-Isaac et al. 2012, Branco et al. 2012). Most of these studies focus on a monopolistic seller's strategic reactions to consumers' partial evaluation. However, because consumers often consider and evaluate several alternatives when shopping at an online intermediary, the study of an intermediary's incentives requires a model to capture sellers' competitive reactions to consumers' partial evaluation. Thus, in this paper, we employ a competitive framework.

The marketing literature has recognized two basic roles of online shopping intermediaries to serve consumers – providing product information of price and non-price attributes (Häubl & Trifts 2000, Iyer & Padmanabhan 2006). Chen et al. (2002) and Iyer & Pazgal (2003) consider the intermediaries' role as disseminators of price information.[3] Chen et al. (2002) study the role of an intermediary as a price discrimination mechanism, while Iyer & Pazgal (2003) focus on the motivation of internet retailers to join an intermediary's service. Like our paper, these studies consider an intermediary that hosts third-party sellers. However, both of these studies focus on situations in which a consumer knows the specific product she wants to buy and uses the intermediary to find the best price. Our paper, in contrast, examines the case in which the consumer uses the intermediary to help her find a good fitting product based on *non-price* attributes. In addition, our paper considers the consumer's evaluation process as a function of the intermediary's designed search environment. Doing so, this research is among the first to incorporate consumers' optimal search behavior into the intermediary's design of the search environment.

Finally, our work is also related to the literature on common agency. Bernheim & Whinston (1985, 1986) show that a common agency may allow sellers to collude and achieve maximal cooperative profit if they delegate the pricing or output decisions to the agency. In contrast, in our paper sellers are assumed to delegate the controls of shopping environment to the intermediary. As we show, although the intermediary cannot help sellers to implement monopoly pricing, it can design the search environment to protect them from fierce competition.

2. Model

There are n third-party sellers, each selling a single product. The products are horizontally differentiated and have no systematic quality difference. The mass of the consumers is normalized to one and each consumer has demand for one product. We assume that the consumer initially has imperfect information about the fit of products' attributes (e.g. color or styling). She must therefore evaluate a product to determine its idiosyncratic utility. Product evaluation is costly to the consumer. But an intermediary provides a search environment to reduce her evaluation costs. Observing such a search environment, but

[3] Jiang et al. (2011), which also studies the price informative role of intermediary, focuses on the potential threat from the intermediary to compete against a third-party seller that incentivizes the seller to mask the popularity of its product from an intermediary.

not products' fit, the consumer chooses how many products to evaluate (evaluation breadth) and how much information to acquire about each considered product (evaluation depth).

These assumptions imply that all the products are *a priori* identical before evaluation, but may differ after evaluation by their *ex post* fit realizations and prices, which affect the consumer's product choice. The timing of the model is as follows. First, the intermediary chooses its search environment (operationalized by choosing search costs). Second, each seller chooses a price for its product. The consumer then selects the evaluation depth (how much information to acquire about a product) and breadth (how many sellers to evaluate). At last, she evaluates products at the selected depth and purchases the best one based on her evaluation.

Because products' *ex post* realized utilities affect the consumer's product choice, we first define the overall *ex post* realized utility of a product (seller i's product) which is observed after full evaluation

$$u_i = v - p_i + \mu\varepsilon_i,$$

where v is a base level of utility,[4] p_i is price, and ε_i is a random utility term. We assume that $\{\varepsilon_i\}$ are i.i.d. and are drawn from a standard extreme value distribution (with cumulative distribution function $e^{-e^{-\varepsilon}}$). The coefficient μ captures the degree of differentiation between products. The random utility of seller i's product $\mu\varepsilon_i$ is a random variable with extreme value distribution (with cumulative distribution function $e^{-e^{-\varepsilon/\mu}}$).

The conventional search literature assumes that by incurring evaluation costs a consumer uncovers the realization of ε_i. However, she cannot observe this overall *ex post* realized utility of a product if she chooses to evaluate it partially. To define the utility of a partially evaluated product, we adopt the following framework. Denote $d\epsilon[0,1]$ as the *evaluation depth* of a given product where $d = 0, 1$ corresponds to no or full evaluation, respectively, and $d\epsilon(0,1)$ means partial evaluation. With the assumed distribution, ε_i has a self-decomposability property,[5] which enables us to define the *ex post* realized utility of a product that is partially evaluated. This property states that for any $d\epsilon[0,1]$, ε_i can be

[4] We assume that v is sufficiently large to induce all consumers to participate in search.
[5] See Steutel & van Harn (1979).

written (equal in distribution) as $\varepsilon_i \overset{D}{=} d\hat{\varepsilon}_i + \theta_i(d)$ where $\hat{\varepsilon}_i$ is a random utility term that is drawn from a standard extreme value distribution, independent from ε_i and $\theta_i(d)$.[6] Thus, if the consumer evaluates product i at depth d, she takes a draw from the random utility $d\hat{\varepsilon}_i$ but does not observe the realized value for the remaining portion $\theta_i(d)$. The random variables $\{d\hat{\varepsilon}_i\}$ are i.i.d. with extreme value distribution (cumulative distribution function $e^{-e^{-\varepsilon/d}}$). Independent of the evaluation depth, we assume that the consumer learns the price of seller i's product, p_i.

We assume that a consumer begins her evaluation of a product with simple fit attributes. For example, when evaluating cameras, a consumer may first look at the brands or sellers' names, followed by sizes and colors, after which she may want to inspect more difficult ones like resolutions and viewfinder types. This suggests that the evaluation cost of a product is convex in evaluation depth (d), which, we assume for simplicity to be quadratic. We denote $\tau(> 0$, referred as to *the baseline evaluation cost*) as the evaluation cost for the consumer to evaluate a product's fit completely, or equivalently, at full depth ($d = 1$). To ensure that the intermediary serves consumers when faced with non-trivial search costs in absence of search environment, we assume that $\tau > \mu e^{-2}$. Thus, the evaluation cost for the consumer to evaluate a product's fit at depth $d \epsilon (0,1]$ is given by $d^2\tau$. It is noteworthy that this evaluation cost is proportional to the resolved uncertainty (measured by the variance of the explained random utility).We assume that a consumer's search process is simultaneous.[7] Before evaluating products, a consumer decides $b \leq n$ sellers at an evaluation depth of $d \epsilon [0,1]$ for each of the b sellers. Let this choice (b, d), which we specify later, be called the *evaluation plan*. With this evaluation plan, the consumer uncovers the realized utility of the evaluated portions $\{d\hat{\varepsilon}_i\}_{i=1}^{b}$ but does not observe the remaining utility $\{\theta_i(d)\}_{i=1}^{b}$. And, among these b partially evaluated products, she chooses the one with the highest expected utility:

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[6] The random variable $\theta(d)$ has Lévy density function $m_d(x) = 0$ for $x < 0$ and $x m_d(x) = \frac{1}{e^{x}-1} - \frac{1}{e^{x/d}-1}$ for $x > 0$ (Steutel & Van Harn, 2003). Note that $\theta(1)$ has a degenerate distribution.

[7] Alternatively, one might model consumer search as a sequential process (Wolinksy 1986). However, a simultaneous search model is sufficient for capturing the basic search incentives without unduly complicating the analysis.

$$i^* = \text{argmax}_{i=1,\ldots,b}\, v - p_i + \mu\{d\hat{\varepsilon}_i + E[\theta_i(d)]\},$$

where $E[\theta_i(d)] = (1-d)\gamma$ denotes the expected value of the unobserved random utility of a product that is evaluated at depth d.[8] Note that this value is independent of i, implying that products differ by their *ex post* fit realizations and prices $\{\mu d\hat{\varepsilon}_i - p_i\}_{i=1}^{b}$. Thus, the best product can be expressed by $i^* = \text{argmax}_{i=1,\ldots,b}\, \mu d\hat{\varepsilon}_i - p_i$. Since $\{d\hat{\varepsilon}_i\}_{i=1}^{b}$ are i.i.d. random variables with extreme value distribution, we can utilize logit model to derive a closed form expression for the choice probability of product i:[9]

$$\mathbb{P}_i = \frac{e^{(v-p_i)/d\mu}}{\sum_{j=1}^{b} e^{(v-p_j)/d\mu}}; \qquad i = 1, \ldots, b.$$

The consumer's evaluation breadth and depth are determined before evaluation. Thus, they depend on the expected evaluation benefits rather than on products' realized utility. By assumption, all products are *a priori* identical before evaluation. Furthermore, the consumer believes (correctly) that all sellers charge the same price in equilibrium: $p_i = p$ for all $i = 1, \ldots, n$. Under this symmetric condition, the expected benefit of evaluation plan (d, b) is given by

$$E(u_{i^*}) = v - p + \mu\{E(d\hat{\varepsilon}_{i^*}) + E[\theta_{i^*}(d)]\},$$

which simplifies to

$$v - p + d\mu \ln(b) + \mu\gamma.$$

We now consider the role of the online shopping intermediary. The intermediary chooses its search environment to control consumers' evaluation costs. Let $s\epsilon[0,1]$ denote the extent the search environment lowers evaluation costs. Then the expected utility from evaluation plan (d, b) in an search environment (s) is given by

$$U(d, b; s) = v - p + d\mu \ln(b) + \mu\gamma - (1-s)b(d^2\tau). \qquad (1)$$

When $s = 0$, the search environment does not make product evaluation any easier and thus the consumer's evaluation costs are not reduced compared to the earlier situation without considering the

[8] $E[\theta_i(d)] = E(\varepsilon_i) - E(d\varepsilon_i) = (1-d)\gamma$, where γ is Euler-Mascheroni constant (≈ 0.5772).

[9] We make the assumption of full market coverage in the model to keep the analysis simple. Allowing a no-buy option does not change the main results. See also section 4 in which consumers have the option to shop at a competing intermediary.

intermediary. Intermediate cases of $s\epsilon(0,1)$ means that the search environment partially lowers her evaluation costs. If $s = 1$, then the search environment makes search so easy that the consumer can costlessly evaluate all sellers' products. The following lemma characterizes the consumer's optimal evaluation depth and breadth as a function of the search environment (s).

Lemma 1: *Let $n > e^2$. For any given s, there exists a unique pair $\left[\hat{d}(s), \hat{b}(s)\right]$ maximizing $U(d, b; s)$ s.t. $\hat{d}(s) \in [0,1]$ and $1 \le \hat{b}(s) \le n$.*

$$\hat{d}(s) = \begin{cases} \frac{\mu}{(1-s)\tau} e^{-2} & 0 \le s < 1 - \frac{\mu}{\tau} e^{-2} \\ 1 & 1 - \frac{\mu}{\tau} e^{-2} \le s \le 1 \end{cases}$$

$$\hat{b}(s) = \begin{cases} e^2 & 0 \le s < 1 - \frac{\mu}{\tau} e^{-2} \\ \frac{\mu}{(1-s)\tau} & 1 - \frac{\mu}{\tau} e^{-2} \le s \le 1 - \frac{\mu}{\tau n}. \\ n & 1 - \frac{\mu}{\tau n} < s \le 1 \end{cases}$$

Furthermore, $\hat{d}'(s) > 0 = \hat{b}'(s)$ for $s \in \left[0, 1 - \frac{\mu}{\tau} e^{-2}\right)$, $\hat{d}'(s) = 0 < \hat{b}'(s)$ for $s \in \left[1 - \frac{\mu}{\tau} e^{-2}, 1 - \frac{\mu}{\tau n}\right]$, and $\hat{d}'(s) = \hat{b}'(s) = 0$ for $s \in \left(1 - \frac{\mu}{\tau n}, 1\right]$.

Lemma 1 characterizes how the consumer chooses her evaluation depth and breadth as a function of the search environment. Specifically, when the search environment modestly lowers search costs ($0 \le s < 1 - \frac{\mu}{\tau} e^{-2}$), the consumer partially evaluates a fixed number of products. And, in this interval, as an environment that makes search easier is offered, her evaluation costs are reduced. This allows her to evaluate these products at a greater depth. As the search environment reaches a threshold ($s = 1 - \frac{\mu}{\tau} e^{-2}$), the consumer engages in full evaluation ($\hat{d}(s) = 1$). And, when s exceeds this threshold, her evaluation costs are further reduced and this allows her to evaluate more products, at a full depth. As even more helpful search environment are provided ($1 - \frac{\mu}{\tau n} < s \le 1$), the consumer fully evaluates all the products available at the intermediary ($\hat{b}(s) = n$). In addition, product differentiation (μ) also influences evaluation depth and breadth. Specifically, larger μ implies larger benefit of evaluation and this implies that the consumer may evaluate more products at a greater depth. It is noteworthy that most online shopping intermediaries provide different search environment (e.g. different sets of search aids) for different products.

A key property of the optimal search plan given in Lemma 1 is the derivative $\hat{b}'(s) = 0$ for small $s \in [0,1 - \mu/e^2\tau]$. In this range, as a more helpful search environment is provided, the consumer increases the depth of her evaluation ($\hat{d}'(s) > 0$) without expanding the breadth. This is due to the fact that for $d < 1$, the consumer's net marginal benefit of depth always exceeds that of breadth. This can formally be seen in how the partials of search objective of (1) with respect to d and b can be influenced by s. When s is small and consumers evaluate partially, a more helpful search environment lowers the marginal cost of depth to a greater extent than that of breadth. Thus, an increase in s induces deeper evaluation without expanding the number of considered sellers. Only when the search environment is sufficiently helpful, $s > 1 - \mu/e^2\tau$, does the further search costs reduction induce the consumer to expand her search breadth: $\hat{b}'(s) > 0$. In this case, the consumer is evaluating products at full depth. This property plays an important role in assessing the intermediary's optimal choice of s in stage 1.

We now consider the game played by the n sellers. The consumer's evaluation plan is made *before* evaluation and thus depends on her rational expectations of the sellers' symmetric equilibrium price. That is, her evaluation plan is not affected by any deviation by a firm from the symmetric equilibrium price. The consumer's product choice, however, depends on such deviations because her purchase decision is determined *after* she observes products' prices. To determine the equilibrium price, we focus on seller i by assuming that it charges price at p_i while all other sellers charge price at p. Under this condition, for any evaluation plan (d,b), we can write the conditional demand for seller i's product (given that it is evaluated) as the choice probability of product i:

$$q_i = \frac{e^{(v-p_i)/d\mu}}{e^{(v-p_i)/d\mu} + (b-1)e^{(v-p_j)/d\mu}}; \qquad i = 1, \dots, b$$

The consumer selects a subset of the n available products on the intermediary to evaluate. Because all the products are *a priori* identical before evaluation, she randomly selects b products and there is a b/n probability for product i being selected for evaluation.[10] Thus, the unconditional demand for seller i's product is $\frac{b}{n}q_i$. Seller i's expected profit is given by:

$$\pi_i = (1-\rho)\frac{b}{n}p_i q_i. \tag{2}$$

where $\rho\epsilon(0,1)$ denotes the referral fee paid to the intermediary. Seller i chooses p_i to maximize its expected profit π_i. The following lemma characterizes sellers' symmetric equilibrium prices given optimal evaluation plan $(\hat{d}(s), \hat{b}(s))$.

Lemma 2: *For any s, with the corresponding optimal evaluation plan $\left[\hat{d}(s), \hat{b}(s)\right]$, sellers' equilibrium prices are given by*

$$\hat{p}(s) = \frac{\hat{d}(s)\mu}{1-\frac{1}{\hat{b}(s)}} = \begin{cases} \frac{1}{e^2-1}\frac{\mu^2}{(1-s)\tau} & 0 \le s < 1-\frac{\mu}{\tau}e^{-2} \\ \frac{\mu^2}{\mu-(1-s)\tau} & 1-\frac{\mu}{\tau}e^{-2} \le s \le 1-\frac{\mu}{\tau n} \\ \frac{\mu n}{n-1} & 1-\frac{\mu}{\tau n} < s \le 1. \end{cases}$$

This lemma illustrates two main points. First, it shows how evaluation depth and breadth affect prices. Specifically, as the consumer evaluates products at a greater depth (larger d), she can better appreciate the value of her most preferred product. This effect, which we call the *evaluation depth effect*, increases sellers' prices in equilibrium. In contrast, as she considers more products (larger b), the competition between sellers pushes prices downwards – an effect we call the *evaluation breadth effect*.

Second, this lemma illustrates how the search environment affects prices through the evaluation depth and breadth. Specifically, when the search environment only modestly lowers search costs ($0 < s < 1-\frac{\mu}{\tau}e^{-2}$), the consumer evaluates products at a greater depth as more helpful environment are

[10] Products can be made more prominent than others on an online shopping intermediary so that consumers may first evaluate the prominent ones. This remains a potential issue for further research.

provided (see Lemma 1). Thus, the evaluation depth effect applies and prices increase. However, when the search environment passes the threshold ($1-\frac{\mu}{\tau}e^{-2} \leq s \leq 1-\frac{\mu}{\tau n}$), the consumer fully evaluates more products as more search costs are lowered. Evaluation breadth effect, therefore, applies and prices decrease. When an even more helpful environment is provided, the consumer fully evaluates all products available at the intermediary. Therefore, larger s affects neither consumers' evaluation plan nor firms' prices.

We now consider the intermediary's design of its search environment. As argued earlier, the intermediary chooses its search environment ($s \in [0,1]$) to control the consumer's evaluation costs. Assume that the intermediary incurs zero cost in providing search environment. The intermediary's expected profit is given by

$$\pi_I = \rho \hat{p}(s) \tag{3}$$

where $\hat{p}(s)$ is the symmetric equilibrium price as given in Lemma 2.

The intermediary's objective in (3) is simply to maximize sellers' equilibrium prices. From Lemma 2, we see $\hat{p}(s)$ is single peaked at $s = 1-\frac{\mu}{\tau}e^{-2}$. Any search environment with s lower than this identified point (or equivalently, any environment with higher evaluation costs) induces the consumer to partially evaluate products, making her unable to fully appreciate the value of her most preferred product. With partial evaluation, sellers cut prices in equilibrium. Alternatively, any search environment with s larger than $1-\frac{\mu}{\tau}e^{-2}$ (or equivalently, any environment with lower evaluation costs) encourages the consumer to broaden the set of sellers evaluated, inducing sellers to price more competitively. The intermediary's optimal search environment reflects, therefore, a balance between sufficiently lowering search costs so that the consumer fully evaluates the considered products and fully knows what she is buying, but not too much that she searches a lot of sellers.

Proposition 1: *In equilibrium, the intermediary has a search environment defined by $s^* = 1-\frac{\mu}{\tau}e^{-2}$ and sellers' prices are given by $\hat{p}(s^*) = \frac{\mu}{1-e^{-2}}$.*

Proposition 1 demonstrates that even though the intermediary designs the search environment with positive evaluation costs, the consumer evaluates sellers' products at full depth. Furthermore, the intermediary sets the search costs in such a way that consumers' "residual" evaluation depth that the consumer evaluates without any help from the search environment is free: $s^* = 1 - \hat{d}(0)$. In other words, suppose the consumer, in the absence of such an environment, would evaluate only a few attributes. The intermediary steps in with search environment to lower search costs so that the consumer evaluates the remaining attributes costlessly. This proposition also illustrates the impact of product differentiation on the intermediary's optimal search environment. Less differentiation implies a lower benefit of product evaluation, which induces the consumer to scale back evaluation depth. To maintain full evaluation, the intermediary reduces search costs by offering a more helpful search environment. However, both the intermediary and sellers hurt by the reduced prices stemming from lower product differentiation.

To gain additional intuition and help understand the insight of the next section, consider an intermediary choosing between two extreme cases: extremely helpful search environment (s = 1) and a search environment that offers no help (s = 0). The intermediary always prefers to provide extremely helpful search environment and this makes search so easy that consumers evaluate all sellers' products at full depth. This earns the intermediary (and its sellers) more profits than if it made no effort to help consumers' search. With a search environment that offers no help, consumers scale back their evaluation depth to such an extent, that products appear to them as less differentiated. Knowing that the consumer sees all products relatively undifferentiated, firms compete more aggressively on prices. This result suggests, therefore, that evaluation depth is perhaps more critical to intermediary profits than evaluation breadth. And, as we shall see in the next section, this insight can help the intermediary when facing heterogeneous consumers with different baseline evaluation costs.

3. Heterogeneous Consumers

The main argument from the above setting suggests that the intermediary uses search environment to choose search costs in order to influence the evaluation depth and breadth of the products consumers consider in order to maintain sellers' prices. The previous insight, however, is built off the assumption that all consumers have the same baseline evaluation costs which reflects that they are homogenous in the product evaluation abilities. In reality, it is likely that some consumers have higher abilities than others due to their superior knowledge in a category. And importantly, intermediaries (e.g. YahooShopping or Taobao Mall) neither customize their design of search environment based on consumers' evaluation abilities nor make it feasible for their sellers to customize their prices. This requires the intermediary to carefully consider the overall effect of its search environment on consumers with different evaluation abilities, which in turn affects sellers' prices. To understand this consideration, we extend the previous setting to the case of consumer heterogeneity in evaluation abilities.

We assume that there are two types of consumers, different in their baseline evaluation costs. Denote τ and $\beta\tau$ $(0 < \beta < 1)$ respectively as the baseline evaluation costs for high-cost consumers and low-cost consumers (henceforth *H*-consumers and *L*-consumers respectively). Smaller β implies larger difference in evaluation abilities between the two consumer types. We assume that the sizes of both types of consumers are equal.[11] In addition, suppose that sellers and the intermediary do not discriminate consumers based on their types. Denote by (d_H, b_H) and (d_L, b_L) as the optimal evaluation plan of *H*-consumers and *L*-consumers, respectively. For a given s, one can apply Lemma 1 to derive (d_H, b_H) and (d_L, b_L) and verify that *L*-consumers will evaluate no fewer products at a no shallower depth ($b_L \geq b_H$, $d_L \geq d_H$) due to their superior evaluation abilities. The following table illustrates optimal evaluation plans for both consumer types based on the search environment.

Search	$s \in \left[0, 1 - \frac{\mu}{e^2\beta\tau}\right)$	$s \in \left[1 - \frac{\mu}{e^2\beta\tau}, 1 - \frac{\mu}{e^2\tau}\right]$	$s \in \left(1 - \frac{\mu}{e^2\tau}, 1\right]$

[11] Equal mass assumption allows us to focus on the effect of consumer heterogeneity on intermediary's search environment design without being influenced by size of the segmentation.

Environment			
Evaluation Depth	$d_H < d_L < 1$	$d_H \leq d_L = 1$	$d_H = d_L = 1$
Evaluation Breadth	$b_H = b_L = e^2$	$b_H = e^2 \leq b_L$	$e^2 < b_H < b_L$
Consumer Evaluation Behavior	Both consumer types partially evaluate products	*H*-consumers partially evaluate products; *L*-consumers fully evaluate products	Both consumer types fully evaluate products

Table 1: Optimal Evaluation Plans for Both Consumer Types

As indicated in Table 1, when the search environment modestly reduces search costs, $s \in \left[0, 1 - \frac{\mu}{e^2\beta\tau}\right)$, both consumer types partially evaluate a constant number of products. Alternatively, when the search environment greatly lowers search costs ($s \in \left(1 - \frac{\mu}{e^2\tau}, 1\right]$), both consumer types fully evaluate more products. Clearly from Lemma 2, neither situation is optimal to the intermediary, implying that the intermediary's profit maximizing search environment falls into the moderate interval $s \in \left[1 - \frac{\mu}{e^2\beta\tau}, 1 - \frac{\mu}{e^2\tau}\right]$. Recall from the previous setting, with homogenous consumers, the optimal search environment is specified simply as the point that maximizes search costs conditional on full evaluation. However, such a condition is impossible to specify in the case of heterogeneous consumers. In this moderate interval, an easier search environment helps *H*-consumers to evaluate products at a greater depth, but at the same time it allows *L*-consumers to fully evaluate more products. That is, greater evaluation depth by *H*-consumers is associated with wider evaluation breadth by *L*-consumers. How the intermediary optimally handles this trade-off is given in the following proposition.

Proposition 2: *If the sizes of consumer types are equal, then the intermediary's profit is maximized at* $s^* = 1 - \frac{\mu}{\tau}e^{-2}$ *so that both L and H-consumers evaluate products at full depth. In addition, L-consumers evaluate* $b_L^* = \frac{e^2}{\beta}$ *sellers when* $\beta \in \left[\frac{e^2}{n}, 1\right)$ *and evaluate* $b_L^* = n$ *sellers when* $\beta \in \left(0, \frac{e^2}{n}\right)$.

This proposition echoes the previous message that greater evaluation depth is more important for the intermediary's profit than narrower evaluation breadth. In equilibrium, the intermediary would rather help *H*-consumers to evaluate products at full depth than limit L-consumers from evaluating more sellers. Thus, the intermediary's design objective should be to err on the side of embedding sufficiently low search costs such that the consumers with lower evaluation abilities can evaluate products at full depth. This result reinforces the importance of full evaluation in the optimality condition identified in Proposition 1. It is important to note that the main design objective to ensure full evaluation depth is so strong that even doing so can allow L-consumers to fully evaluate all the sellers available on the intermediary when their evaluation abilities are very high ($\beta \in (0, e^2/n)$).

4. Intermediary Competition

The previous section studies how one intermediary optimally designs its search environment. Because a more helpful search environment reduces evaluation costs, an intermediary can use them as a means to attract consumers from its rivals. Therefore, the design of search environment requires an intermediary to consider consumer demand, in addition to protecting sellers from fierce competition. In this section, we study how intermediary competition affects the strategic design of the search environment. We ask, in particular, how the optimal search environment condition found in Proposition 1 is affected by competition among intermediaries.

Consider a market with two competing intermediaries (denoted by $j = 1, 2$), each with n sellers. We assume that the consumer has individual preferences over both intermediaries (e.g. horizontal features that distinguish both intermediaries) and only chooses one intermediary at which to shop. All other assumptions about consumers are the same as in section 2. This set up implies that the consumer's intermediary choice depends on her expected surplus of evaluating and buying at each intermediary, which includes two factors: expected net benefit of product evaluation and her individual preference for an intermediary.

The timing of the model is as follows. First, intermediaries simultaneously choose their search environments. Second, each seller chooses a price for its product. Third, the consumer chooses an intermediary. She then selects evaluation depth and breadth. As before, the choices of depth and breadth are simultaneous. At last, she evaluates products at the selected depth and purchases the best one based on her evaluation. This timing implies that given a chosen intermediary, the consumer chooses evaluation depth and breadth and selects the best product in the same way as in section 2, as given in Lemma 1. We therefore start our analysis from stage three where the consumer chooses one intermediary at which to shop.

The consumer's intermediary choice depends on her expected net benefit of product evaluation at each intermediary,

$$U_j(d_j, b_j; s_j) = v - p_j + d_j\mu \ln(b_j) - (1 - s_j)b_j({d_j}^2\tau)$$

where s_j is the search environment on intermediary j, (d_j, b_j) is the consumer's evaluation plan on intermediary j which can be obtained from Lemma 1, and p_j is the equilibrium price on intermediary j. As before, we treat sellers on an intermediary as *a priori* symmetric and therefore focus on equilibria where all the sellers on a given intermediary charge the same price. Because the consumer chooses an intermediary before evaluation, her intermediary choice depends on her rational expectations of the equilibrium prices.

To capture the differentiation among intermediaries, we assume that consumers (unit mass with unit demand) are uniformly distributed on a standard Hotelling line of unit length, with two intermediaries located at each extreme (without loss of generality, intermediary 1 and 2 are respectively located at location 0 and 1). Horizontal differentiation between the two intermediaries may reflect, for example, website style and layout. A consumer's location on the line reveals her ideal preference and she has disutility if her location does not match that of the chosen intermediary. Let t denote the linear travel cost per location unit traveled. Larger t implies greater heterogeneity in consumers' tastes towards the intermediaries (or one can view t as the indicator of intermediaries' horizontal differentiation). Thus, the consumer located at $x \epsilon [0,1]$ has total disutility tx^2 for intermediary 1 and $t(1-x)^2$ for intermediary 2.

Thus, this consumer's expected surplus of evaluating and buying at intermediary 1 and 2 are respectively given by

$$U_1(d_1, b_1; s_1) - tx^2 \tag{4a}$$

$$U_2(d_2, b_2; s_2) - t(1-x)^2 \tag{4b}$$

Assume that the consumer chooses the intermediary which gives her larger expected surplus.[12]

To derive consumer demand for each intermediary, suppose that the consumer located at $\bar{x}$ is indifferent between the two intermediaries and the market coverage is full, $U_1(d_1, b_1; s_1) - t\bar{x}^2 = U_2(d_2, b_2; s_2) - t(1-\bar{x})^2$. This implies that consumers located at $x \epsilon [0, \bar{x})$ choose intermediary 1 because (4a) is larger and consumers located at $x \epsilon (\bar{x}, 1]$ choose intermediary 2 because (4b) is larger. Let D_j denote consumer demand for intermediary j. Consumer demand for each intermediary is given by

$$D_1 = \bar{x} = \frac{U_1(d_1,b_1;s_1) - U_2(d_2,b_2;s_2)}{2t} + \frac{1}{2} \text{ and } D_2 = 1 - \bar{x} = \frac{U_2(d_2,b_2;s_2) - U_1(d_1,b_1;s_1)}{2t} + \frac{1}{2}. \tag{5}$$

We now consider the game played by the n competing sellers of each intermediary. A seller's profit is determined by its price, and also by the overall consumer demand at its intermediary which was previously assumed to be fixed in section 2. To determine the symmetric equilibrium prices, we focus on seller i of intermediary j by assuming that it sets price at p_{ij} and other sellers at this intermediary sets price at $p_{i'j}$. The expected profit of seller i is given by

$$\pi_{ij} = (1-\rho) D_j \left(\frac{b_j}{n}\right) p_{ij} \frac{e^{(v-p_{ij})/d_j\mu}}{e^{(v-p_{ij})/d_j\mu} + (b_j - 1) e^{(v-p_{i'j})/d_j\mu}} \tag{6}$$

where $\rho \epsilon (0,1)$ denotes the referral fees charged by intermediary j. Two points about (6) should be emphasized. First, the only difference between expressions (6) and (2) is the extra term D_j, capturing the consumer traffic at intermediary j. Second, as argued earlier, D_j is affected by the equilibrium prices (see

[12] We assume that v is sufficiently large so that either (4a) or (4b) is positive for any x. This ensures that (1) all consumers are induced to participate in search and (2) competition between intermediaries is maintained for large travel costs.

(5)). That is, any seller's deviation from a potential symmetric equilibrium cannot affect D_j. Thus, seller i on intermediary j does not internalize D_j when choosing its price p_{ij}, implying that travel cost (t) does not directly affect seller i's choice of price. These two points also imply that sellers' prices at a given intermediary are identical as given in Lemma 2.

Intermediary j chooses its search environment s_j to maximize its expected profit:

$$\pi_j = \rho D_j p_j. \quad (7)$$

It is important to note that intermediary j internalizes its overall consumer demand D_j when choosing s_j, which is the main distinction from section 2. Thus, travel cost affects equilibrium prices only through intermediaries' incentives, a result established in the following proposition.

Proposition 3:

(i) *When intermediaries are relatively differentiated* ($t/\mu > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$)*, the equilibrium outcomes are the same as in Proposition 1.*

(ii) *Otherwise, in equilibrium, the intermediary minimizes search costs in the search environment* ($s^* = 1$). *The symmetric equilibrium price is* $\mu\left(\frac{n}{n-1}\right)$.

This proposition illustrates that when intermediaries are relatively more differentiated than sellers, large t/μ, they do not have to aggressively compete for consumers by providing very helpful search environment that greatly lowers search costs. This allows intermediaries to focus on protecting sellers from fierce competition just as the monopoly intermediary does in section 2 and we obtain the same outcome as indicated in Proposition 1. Alternatively, when intermediaries are relatively less differentiated than sellers, low t/μ, they compete more aggressively to attract consumers by providing very helpful search environments, although doing so intensifies competition among sellers. In fact, in this case, because $s^* = 1$ consumers incur no search costs and evaluate all sellers at the chosen intermediary. Therefore, unlike the monopoly case, the number of sellers n affects the equilibrium prices.

The number of sellers on an intermediary also plays a role in determining the search environment through its impact on the threshold for t/μ. As indicated in Proposition 3, this threshold is U-shaped in n. For intermediate levels of n, intermediaries provide the monopoly search environment ($s^* < 1$). When the number of sellers is very small or very large, however, intermediaries have a greater incentive to provide perfectly helpful search environment ($s^* = 1$). Specifically, as n becomes very small, consumers optimally evaluate all sellers, even with positive search costs (See Lemma 1). By removing search costs, a competitive intermediary attracts additional consumers without adding competitive pressure on sellers' prices. On the other hand, when n becomes very large, intermediaries enter a prisoner's dilemma. That is, by jointly maintaining $s^* < 1$, intermediaries earn higher profits. But, unilaterally lowering search costs, an intermediary can attract consumers to its platform by ensuring them more surplus (better fitting seller at reduced search costs).

5. Conclusion

This paper has examined strategic design considerations of online shopping intermediaries' search environments. We defined a search environment as means to control consumers' evaluation costs. Because these intermediaries receive revenue from competing third-party sellers, they must balance improvements in consumers' search benefits with sellers' profit incentives. We showed that an intermediary's optimal search environment includes positive search costs, but only up to a point. If consumers face too much search costs, they can scale back the depth of their evaluation of sellers' products – a regime of partial product evaluation. Our model indicated that it is optimal for the intermediary to provide search environment that embeds sufficiently low evaluation costs to ensure consumers evaluate products at full depth. In addition, our findings showed that for heterogeneous consumers the intermediary should provide helpful enough search environment to just guarantee all consumers (including the consumers with low evaluation abilities) fully evaluate products, a result reinforcing the importance of full evaluation in the intermediary's design objective. We also showed that competitive intermediaries can be induced to provide more helpful search environment in order to attract shoppers to their platforms, but only if they are sufficiently undifferentiated.

Our research is only a first attempt at understanding the strategic factors that affect the design of the search environment at online shopping intermediaries. We focused on the strategic role that consumer search costs play in consumers' ability to find the right product and third-party sellers' ability to set profitable prices. In so doing, we omitted several strategic variables that intermediaries may consider. One such factor is the type of third-party sellers the intermediary allows to sell on its platform. We focused on symmetric sellers of identical *ex ante* quality. Given that consumers typically differ in the taste for quality, shopping intermediaries may find it profitable to provide a search environment with a set of vertically differentiated sellers. Another interesting factor in the design of the search environment arises when third-party sellers provide multiple products. In fact, most of these sellers on intermediaries carry several products, trying to seek better fits to consumers' idiosyncratic tastes. This provokes the possibility that consumers' search decisions involve two dimensions – across firm and within firm (Liu & Dukes 2013). Because sellers do not compete in price among their own products, search costs may play a different role for intermediary profitability. In addition, it might be interesting to study intermediary's strategic consideration of search environment design in price dimension, especially in the situation where there are many sellers selling the identical products. We believe research on these issues can help us better understand the design of search environments at online shopping intermediaries.

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Appendix (Proofs of Propositions and Lemma)

Proof of Lemma 1

Consumers simultaneously choose the optimal evaluation breadth and depth by maximizing (1) with respect to $b \leq n$ and $d \in [0, 1]$.[13] The first order condition yields the expressions for $\hat{b}(s)$ and $\hat{d}(s)$ in Lemma 1. Checking the Hessian matrix:

$$\frac{\partial^2 U(d,b;s)}{\partial b^2} = -\frac{d\mu}{b^2} = \begin{cases} -\frac{\mu^2}{(1-s)\tau} e^{-6} & 0 \leq s < 1 - \frac{\mu}{\tau} e^{-2} \\ -\frac{(1-s)^2\tau^2}{\mu} & 1 - \frac{\mu}{\tau} e^{-2} \leq s \leq 1 - \frac{\mu}{\tau n} \\ -\frac{\mu}{n^2} & 1 - \frac{\mu}{\tau n} < s \leq 1 \end{cases},$$

[13] Liu & Dukes (2014) show that symmetric depth *d* is an optimal solution in the setting in which consumers are permitted to evaluate products with different depths.

$$\frac{\partial^2 U(d,b;s)}{\partial d^2} = -2(1-s)b\tau = \begin{cases} -2(1-s)e^2\tau & 0 \le s < 1-\frac{\mu}{\tau}e^{-2} \\ -2\mu & 1-\frac{\mu}{\tau}e^{-2} \le s \le 1-\frac{\mu}{\tau n}, \\ -2(1-s)n\tau & 1-\frac{\mu}{\tau n} < s \le 1 \end{cases}$$

$$\frac{\partial^2 U(d,b;s)}{\partial b \partial d} = \frac{\mu}{b} - 2(1-s)d\tau = \begin{cases} -\frac{\mu}{e^2} & 0 \le s < 1-\frac{\mu}{\tau}e^{-2} \\ -(1-s)\tau & 1-\frac{\mu}{\tau}e^{-2} \le s \le 1-\frac{\mu}{\tau n}. \\ \frac{\mu}{n} - 2(1-s)\tau & 1-\frac{\mu}{\tau n} < s \le 1 \end{cases}$$

One can easily verify that $\frac{\partial^2 U(d,b;s)}{\partial b^2}\frac{\partial^2 U(d,b;s)}{\partial d^2} - \left[\frac{\partial^2 U(d,b;s)}{\partial b \partial d}\right]^2 > 0$ holds for $0 \le s \le 1-\frac{\mu}{\tau n}$. Hence, the solution to $\frac{\partial U}{\partial b} = \frac{\partial U}{\partial d} = 0$ is the interior maximum for $s \in \left[0, 1-\frac{\mu}{\tau n}\right]$.

Now suppose $1-\frac{\mu}{\tau n} \le s \le 1$. Then

$$\left.\frac{\partial U}{\partial d}\right|_{b=e^2} = 2\mu[1-(1-s)e^2\tau d/\mu] > 2\mu\left(1-\frac{e^2}{n}d\right),$$

which is positive for all $d \in [0,1]$ and $n > e^2$. Therefore, setting $d = 1$ is required at any maximized solution. In this case, the consumer optimal choice of $b \le n$ must satisfy either $\left.\frac{\partial U}{\partial b}\right|_{d=1,b<n} = 0$ or $\left.\frac{\partial U}{\partial b}\right|_{d=1,b=n} \ge 0$. In the first case, we must have $b = \frac{\mu}{(1-s)\tau}$. This solution for b is a maximizer since $\left.\frac{\partial^2 U}{\partial b^2}\right|_{d=1,b=\frac{\mu}{(1-s)\tau}} < 0$. In the second case, at the boundary $b = n$, we have $\left.\frac{\partial U}{\partial b}\right|_{d=1,b=n} = \frac{\mu}{n} - (1-s)\tau \ge 0$ under the condition that $s \ge 1-\frac{\mu}{\tau n}$. Hence, the boundary solutions $\left(d = 1, b = \frac{\mu}{(1-s)\tau}\right)$ and $(d = 1, b = n)$ are both maximizing. ■

Proof of Lemma 2

Given $[\hat{d}(s), \hat{b}(s)]$, we determine equilibrium price by maximizing (2) with respect to p_i and invoking symmetry. It is straightforward to show

$$\frac{\partial \pi_i}{\partial p_i} = (1-\rho)\frac{b}{n}\left[p_i \frac{q_i(q_i-1)}{d\mu} + q_i\right] = 0,$$

implies $\hat{p}(s) = \frac{\hat{d}(s)\mu}{1-\frac{1}{\hat{b}(s)}}$. Substituting in the expressions for $\hat{d}(s)$ and $\hat{b}(s)$ into $\hat{p}(s)$yields the expression for prices given in the statement of the lemma.

One can also check

$$\frac{\partial^2 \pi_i}{\partial {p_i}^2} = (1-\rho)\frac{b}{n}\left[p_i \frac{q_i(q_i-1)(2q_i-1)}{(d\mu)^2} + \frac{2q_i(q_i-1)}{d\mu}\right].$$

Evaluating $\frac{\partial^2 \pi_i}{\partial {p_i}^2}$ at any point where $\frac{\partial \pi_i}{\partial p_i} = 0$ yields

$$\frac{\partial^2 \pi_i}{\partial {p_i}^2} = -(1-\rho)\frac{b}{n}\frac{q_i}{d\mu} < 0.$$

Thus, $p_i = \hat{p}(s)$ satisfies the seller's S.O.C. for profit maximization.■

Proof of Proposition 1

The intermediary chooses s to maximize (3) subject to $s \in [0,1]$. That is, the profit maximizing level of search aids is obtained by maximizing sellers' price $\hat{p}(s)$. From the expression for $\hat{p}(s)$ in Lemma 2, $\hat{p}(s)$ increases in s for $s \in \left[0, 1-\frac{\mu}{\tau}e^{-2}\right)$, decreases in s for $s \in \left[1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{\tau n}\right]$ and is independent from s for $s \in (1-\frac{\mu}{\tau n}, 1]$. This yields the profit maximizing level of search aids s^* as expressed in the statement of the proposition. Plugging s^* back into $\left[\hat{d}(s), \hat{b}(s)\right]$ and $\hat{p}(s)$ yields the result. ■

Proof of Proposition 2

We first determine the equilibrium prices of sellers. Assume that seller i sets price at p_i while all other sellers set price at p. Under this condition, seller i's expected profit is given by,

$$\pi_i = (1-\rho)\left[\frac{1}{2}\frac{b_H}{n}p_i\frac{e^{\frac{v-p_i}{d_H\mu}}}{e^{\frac{v-p_i}{d_H\mu}}+(b_H-1)e^{\frac{v-p}{d_H\mu}}} + \frac{1}{2}\frac{b_L}{n}p_i\frac{e^{\frac{v-p_i}{d_L\mu}}}{e^{\frac{v-p_i}{d_L\mu}}+(b_L-1)e^{\frac{v-p}{d_L\mu}}}\right],$$

Taking the FOC of π_i with respect to p_i and invoking symmetry yields the prices $p = \frac{2\mu}{\frac{b_L-1}{b_L d_L}+\frac{b_H-1}{b_H d_H}}$.

Following a proof similar to that of Lemma 2, one can check that

$$\frac{\partial \pi_i}{\partial p_i} = \frac{1-\rho}{2}\left\{\frac{b_H}{n}\left[p_i\frac{q_{Hi}(q_{Hi}-1)}{d_H\mu} + q_{Hi}\right] + \frac{b_L}{n}\left[p_i\frac{q_{Li}(q_{Li}-1)}{d_L\mu} + q_{Li}\right]\right\},$$

where $q_{Hi} = \frac{e^{\frac{v-p_i}{d_H\mu}}}{e^{\frac{v-p_i}{d_H\mu}}+(b_H-1)e^{\frac{v-p}{d_H\mu}}}$ and $q_{Li} = \frac{e^{\frac{v-p_i}{d_L\mu}}}{e^{\frac{v-p_i}{d_L\mu}}+(b_L-1)e^{\frac{v-p}{d_L\mu}}}$.

Evaluating $\frac{\partial^2 \pi_i}{\partial {p_i}^2}$ at $p = \frac{2\mu}{\frac{b_L-1}{b_L d_L}+\frac{b_H-1}{b_H d_H}}$ yields

$$\frac{\partial^2 \pi_i}{\partial {p_i}^2} = \frac{1-\rho}{n}\left[\frac{\frac{1}{b_H}+\frac{b_L-1}{b_L d_L}d_H}{\frac{b_L-1}{b_L d_L}d_H+\frac{b_H-1}{b_H}}\frac{1}{d_H\mu}\left(\frac{1}{b_H}-1\right) + \frac{\frac{1}{b_L}+\frac{b_H-1}{b_H d_H}d_L}{\frac{b_H-1}{b_H d_H}d_L+\frac{b_L-1}{b_L}}\frac{1}{d_L\mu}\left(\frac{1}{b_L}-1\right)\right],$$

which is negative because $b_L \geq b_H > 1$. Thus, the solution to the F.O.C. is a maximizer.

Next, we determine the consumers' evaluation plan (d_H, b_H) and (d_L, b_L) for each interval illustrated in Table 1.

When $\beta n \geq e^2$,

(i) For $s \in \left[0, 1 - \frac{\mu}{e^2\beta\tau}\right)$, one can verify from Table 1 that the evaluation depth and breadth for both types of consumers are respectively given by $\left(d_L = \frac{\mu}{e^2\beta(1-s)\tau}, b_L = e^2\right)$ and $\left(d_H = \frac{\mu}{e^2(1-s)\tau}, b_H = e^2\right)$.

(ii) For $s \in \left[1 - \frac{\mu}{e^2\beta\tau}, 1 - \frac{\mu}{e^2\tau}\right)$, the evaluation plan for both types of consumers are respectively given by $\left(d_L = 1, b_L = \frac{\mu}{(1-s)\beta\tau}\right)$ and $\left(d_H = \frac{\mu}{e^2(1-s)\tau}, b_H = e^2\right)$.

(iii) For $s \in \left[1 - \frac{\mu}{e^2\tau}, 1 - \frac{\mu}{\beta\tau n}\right)$, the evaluation plan for both types of consumers are respectively given by $\left(d_L = 1, b_L = \frac{\mu}{(1-s)\beta\tau}\right)$ and $\left(d_H = 1, b_H = \frac{\mu}{(1-s)\tau}\right)$.

(iv) For $s \in \left[1 - \frac{\mu}{\beta\tau n}, 1 - \frac{\mu}{\tau n}\right)$, the evaluation plan for both types of consumers are respectively given by $(d_L = 1, b_L = n)$ and $\left(d_H = 1, b_H = \frac{\mu}{(1-s)\tau}\right)$.

(v) For $s \in \left[1 - \frac{\mu}{\tau n}, 1\right]$, the evaluation plan for both types of consumers are respectively given by $(d_L = 1, b_L = n)$ and $(d_H = 1, b_H = n)$.

Plugging (d_H, b_H) and (d_L, b_L) back into p yields

$$p = \begin{cases} \frac{2\mu^2}{(e^2-1)(1-s)\tau(\beta+1)} & s \in \left[0, 1 - \frac{\mu}{e^2\beta\tau}\right) \\ \frac{2\mu^2}{\mu + (e^2-1-\beta)(1-s)\tau} & s \in \left[1 - \frac{\mu}{e^2\beta\tau}, 1 - \frac{\mu}{e^2\tau}\right) \\ \frac{\mu^2}{\mu - \frac{(\beta+1)(1-s)\tau}{2}} & s \in \left[1 - \frac{\mu}{e^2\tau}, 1 - \frac{\mu}{\beta\tau n}\right) \\ \frac{2\mu^2}{\frac{2n-1}{n}\mu - (1-s)\tau} & s \in \left[1 - \frac{\mu}{\beta\tau n}, 1 - \frac{\mu}{\tau n}\right) \\ \frac{\mu n}{n-1} & s \in \left[1 - \frac{\mu}{\tau n}, 1\right], \end{cases}$$

which increases in $s \in \left[0,1 - \frac{\mu}{e^2\tau}\right]$, decreases in $s \in \left(1 - \frac{\mu}{e^2\tau}, 1 - \frac{\mu}{\tau n}\right)$ and is independent of $s \in \left[1 - \frac{\mu}{\tau n}, 1\right]$. This yields the intermediary's profit maximizing level of search aids in search environment,

$$s^* = 1 - \frac{\mu}{e^2\tau}.$$

Plugging s^* back into the expressions for (d_H, b_H) and (d_L, b_L)yields

$$d_H^* = 1,\ b_H^* = e^2 \text{ and } d_L^* = 1,\ b_L^* = \frac{e^2}{\beta} > e^2.$$

When $\beta n < e^2$,

(i) For $s \in \left[0, 1 - \frac{\mu}{e^2\beta\tau}\right)$, one can verify from Table 1 that the evaluation depth and breadth for both types of consumers are respectively given by $\left(d_L = \frac{\mu}{e^2\beta(1-s)\tau}, b_L = e^2\right)$ and $\left(d_H = \frac{\mu}{e^2(1-s)\tau}, b_H = e^2\right)$.

(ii) For $s \in \left[1-\frac{\mu}{e^2\beta\tau}, 1-\frac{\mu}{\beta\tau n}\right)$, the evaluation plan for both types of consumers are respectively given by $\left(d_L = 1, b_L = \frac{\mu}{(1-s)\beta\tau}\right)$ and $\left(d_H = \frac{\mu}{e^2(1-s)\tau}, b_H = e^2\right)$.

(iii) For $s \in \left[1-\frac{\mu}{\beta\tau n}, 1-\frac{\mu}{e^2\tau}\right)$, the evaluation plan for both types of consumers are respectively given by $(d_L = 1, b_L = n)$ and $\left(d_H = \frac{\mu}{e^2(1-s)\tau}, b_H = e^2\right)$.

(iv) For $s \in \left[1-\frac{\mu}{e^2\tau}, 1-\frac{\mu}{\tau n}\right)$, the evaluation plan for both types of consumers are respectively given by $(d_L = 1, b_L = n)$ and $\left(d_H = 1, b_H = \frac{\mu}{(1-s)\tau}\right)$.

(v) For $s \in \left[1-\frac{\mu}{\tau n}, 1\right]$, the evaluation plan for both types of consumers are respectively given by $(d_L = 1, b_L = n)$ and $(d_H = 1, b_H = n)$.

Plugging (d_H, b_H) and (d_L, b_L) back into p yields

$$p = \begin{cases} \frac{2\mu^2}{(e^2-1)(1-s)\tau(\beta+1)} & s \in \left[0, 1-\frac{\mu}{e^2\beta\tau}\right) \\ \frac{2\mu^2}{\mu+(e^2-1-\beta)(1-s)\tau} & s \in \left[1-\frac{\mu}{e^2\beta\tau}, 1-\frac{\mu}{\beta\tau n}\right) \\ \frac{2\mu^2}{\frac{n-1}{n}\mu+(e^2-1)(1-s)\tau} & s \in \left[1-\frac{\mu}{\beta\tau n}, 1-\frac{\mu}{e^2\tau}\right) \\ \frac{2\mu^2}{\frac{2n-1}{n}\mu-(1-s)\tau} & s \in \left[1-\frac{\mu}{e^2\tau}, 1-\frac{\mu}{\tau n}\right) \\ \frac{\mu n}{n-1} & s \in \left[1-\frac{\mu}{\tau n}, 1\right] \end{cases},$$

which increases in $s \in \left[0, 1-\frac{\mu}{e^2\tau}\right]$, decreases in $s \in (1-\frac{\mu}{e^2\tau}, 1-\frac{\mu}{\tau n})$ and is independent of $s \in \left[1-\frac{\mu}{\tau n}, 1\right]$. This yields the intermediary's profit maximizing level of search aids in search environment,

$$s^* = 1-\frac{\mu}{e^2\tau}.$$

Plugging s^* back into the expressions for (d_H, b_H) and (d_L, b_L)yields

$$d_H^* = 1,\ b_H^* = e^2 \text{ and } d_L^* = 1,\ b_L^* = n > e^2.\blacksquare$$

Proof of Proposition 3

We claim that there is a symmetric equilibrium in which both intermediaries set

$$s_j^* = \begin{cases} 1-\frac{\mu}{\tau}e^{-2} & \frac{t}{\mu} > \frac{n(e^2-1)}{n-e^2}\left[\ln(n)-\frac{n}{n-1}+\frac{1}{e^2-1}\right] \\ 1 & 0 \le \frac{t}{\mu} \le \frac{n(e^2-1)}{n-e^2}\left[\ln(n)-\frac{n}{n-1}+\frac{1}{e^2-1}\right] \end{cases}.$$

To prove this claim we demonstrate directly that one intermediary, intermediary 1, cannot be more profitable by deviating from s_1^* given that the other intermediary, intermediary 2, chooses s_2^*.

(i) Suppose $\frac{t}{\mu} > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$ and consider any deviation $s_1 \neq s_1^* = s_2^* = 1 - \frac{\mu}{\tau}e^{-2}$, with the corresponding profits denoted by $\tilde{\pi}_1(s_1)$. Any deviation $s_1 \in \left[0, 1 - \frac{\mu}{\tau}e^{-2}\right)$ leads to profits

$$\tilde{\pi}_1(s_1) = \frac{1}{2t}\left(\frac{1}{e^2-1}\right)\frac{\rho\mu^2}{(1-s_1)\tau}\left[t - \frac{1}{e^2(e^2-1)}\frac{\mu^2}{(1-s_1)\tau} + \frac{\mu}{e^2-1}\right],$$

which we show is increasing on this interval. Specifically, $\frac{\partial\tilde{\pi}_1}{\partial s_1} > 0$ as long as

$$\frac{t}{\mu} > \frac{2\mu}{e^2(e^2-1)(1-s_1)\tau} - \frac{1}{e^2-1}.$$

for all s_1. We have,

$$\frac{t}{\mu} > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right] > \frac{1}{e^2-1} > \frac{2\mu}{e^2(e^2-1)(1-s_1)\tau} - \frac{1}{e^2-1}.$$

where the first inequality holds by assumption, the second since $n > e^2$, and the third for $s_1 \in \left[0, 1 - \frac{\mu}{\tau}e^{-2}\right)$. Therefore, any deviation $s_1 < s_1^* = 1 - \frac{\mu}{\tau}e^{-2}$ is not profitable.

Any deviation $s_1 \in \left(1 - \frac{\mu}{\tau}e^{-2}, 1 - \frac{\mu}{n\tau}\right)$ leads to

$$\tilde{\pi}_1(s_1) = \left(\frac{1}{2t}\right)\frac{\rho\mu^2}{\mu-(1-s_1)\tau}\left\{t - \frac{\mu^2}{\mu-(1-s_1)\tau} + \mu\ln\left[\frac{\mu}{(1-s_1)\tau}\right] - \mu - \left[-\frac{\mu}{1-\frac{1}{e^2}} + \mu\right]\right\}.$$

This deviation is not profitable if $\frac{\partial\tilde{\pi}_1}{\partial s_1} < 0$, which requires

$$\frac{t}{\mu} > f(s_1) \equiv \frac{\mu}{(1-s_1)\tau} + \frac{2\mu}{\mu-(1-s_1)\tau} - \ln\left[\frac{\mu}{(1-s_1)\tau}\right] - \frac{1}{e^2-1}.$$

Since $\frac{t}{\mu} > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right] > e^2 + \frac{1}{e^2-1} = f\left(1 - \frac{\mu}{\tau}e^{-2}\right)$, profits are decreasing near (and to the right of) s_1^*. Note that the function $f(s_1)$ is strictly increasing in s_1. Therefore, the condition $\pi_1^* > \tilde{\pi}(s_1)$ at $s_1 = 1 - \frac{\mu}{n\tau}$, the right endpoint of the interval, is sufficient for $\pi_1^* > \tilde{\pi}(s_1)$ at any $s_1 \in \left(1 - \frac{\mu}{\tau}e^{-2}, 1 - \frac{\mu}{n\tau}\right)$. This condition is

$$\pi_1^* = \frac{\rho\mu}{2}\left(\frac{e^2}{e^2-1}\right) > \left(\frac{\rho\mu}{2t}\right)\left(\frac{n}{n-1}\right)\left[t + \mu\ln(n) - \frac{n}{n-1}\mu + \left(\frac{e^2}{e^2-1}\right)\mu - 2\mu\right] = \tilde{\pi}_1\left(1 - \frac{\mu}{n\tau}\right),$$

which holds by our assumption $\frac{t}{\mu} > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$.

Any deviation $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right]$ leads to a profit of

$$\tilde{\pi}_1(s_1) = \left(\frac{\rho\mu}{2t}\right)\left(\frac{n}{n-1}\right)\left[t + \mu\ln(n) - \frac{n}{n-1}\mu - (1-s_1)n\tau + \frac{\mu}{e^2-1}\right],$$

which is increasing in s_1 and therefore bounded above by $\tilde{\pi}(s_1 = 1)$. The condition $\frac{t}{\mu} > \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$ directly implies that

$$\pi_1^* = \frac{\rho\mu}{2}\left(\frac{e^2}{e^2-1}\right) > \left(\frac{\rho\mu}{2t}\right)\left(\frac{n}{n-1}\right)\ \left[t + \mu\ln(n) - \frac{n}{n-1}\mu + \frac{\mu}{e^2-1}\right] = \tilde{\pi}_1(1) \geq \tilde{\pi}_1(s_1),$$

for all $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right]$.

(ii) Now suppose $\frac{t}{\mu} \leq \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$. We first consider any deviation $s_1 \in \left[0, 1 - \frac{\mu}{\tau}\mathrm{e}^{-2}\right)$. This leads to a deviation profit of

$$\tilde{\pi}_1(s_1) = \frac{\rho\mu^2}{2t\tau}\left[\frac{1}{(1-s_1)(e^2-1)}\right]\left\{t - \frac{\mu^2}{\tau e^2}\left[\frac{1}{(1-s_1)(e^2-1)}\right] + \mu\left[\frac{n}{n-1} - \ln(n)\right]\right\}.$$

Characterizing the shape of this deviation profit function depends on the relative level of t/μ. We argue that $\tilde{\pi}_1(s_1) \leq \pi_1^*$ for different three levels of t/μ.

For $0 < \frac{t}{\mu} < \ln(n) - \frac{n}{n-1}$, the expression for the demand at intermediary 1,

$$\widetilde{D}_1 = \frac{1}{2t}\left\{t - \frac{\mu^2}{\tau e^2}\frac{1}{(1-s_1)(e^2-1)} + \mu\left[\frac{n}{n-1} - \ln(n)\right]\right\} < 0.$$

So any deviation under this condition is not profitable.

For $\ln(n) - \frac{n}{n-1} \leq \frac{t}{\mu} \leq \ln(n) - \frac{n}{n-1} + \frac{2}{e^2-1}$, the derivative $\partial\tilde{\pi}_1/\partial s_1$ has the following property:

$$\left.\frac{\partial\tilde{\pi}_1}{\partial s_1}\right|_{s_1 = 1 - \frac{\mu}{\tau}e^{-2}} < 0 < \left.\frac{\partial\tilde{\pi}_1}{\partial s_1}\right|_{s_1 = 0}.$$

Since the derivative is continuous, it means that any maximizer, $\hat{s}_1$, of $\tilde{\pi}_1(s_1)$ in $\left[0, 1 - \frac{\mu}{\tau}\mathrm{e}^{-2}\right)$ must solve $\frac{\partial\tilde{\pi}_1}{\partial s_1} = 0$. This solution is expressed $\hat{s}_1 = 1 - \frac{2\mu}{\tau}\left\{e^2(e^2-1)\left[\frac{t}{\mu} - \ln(n) + \frac{n}{n-1}\right]\right\}^{-1}$ and leads to profits

$$\tilde{\pi}_1(\hat{s}_1) = \left(\frac{e^2\rho\mu}{8}\right)\left[\frac{t}{\mu} - \ln(n) + \frac{n}{n-1}\right]\left\{1 - \frac{\mu}{t}\left[\ln(n) - \frac{n}{n-1}\right]\right\}.$$

Under the condition that $\frac{t}{\mu} \leq \ln(n) - \frac{n}{n-1} + \frac{2}{e^2-1}$,

$$\tilde{\pi}_1(\hat{s}_1) \leq \frac{1}{4}\left(\frac{e^2\rho\mu}{e^2-1}\right)\left(\frac{\frac{2}{e^2-1}}{\ln(n) - \frac{n}{n-1} + \frac{2}{e^2-1}}\right) < \frac{\rho\mu}{2(1-1/n)},$$

where the last term is the profit π_1^* the intermediary earns by sticking to $s_1^* = 1$. Hence, no deviation $s_1 \in \left[0, 1 - \frac{\mu}{\tau}e^{-2}\right)$ is profitable.

For $\ln(n) - \frac{n}{n-1} + \frac{2}{e^2-1} < \frac{t}{\mu} \le \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$, we have $\frac{\partial\tilde{\pi}_1}{\partial s_1} > 0$ for all $s_1 \in \left[0,1-\frac{\mu}{\tau}\mathrm{e}^{-2}\right)$. Thus,

$$\tilde{\pi}_1(s_1) \le \tilde{\pi}_1\left(1-\frac{\mu}{\tau}e^{-2}\right) = \frac{\rho\mu}{2t}\left(\frac{e^2}{e^2-1}\right)\left\{t - \mu\left[\frac{1}{e^2-1} - \frac{n}{n-1} + \ln(n)\right]\right\} \le \pi_1^*,$$

for all $s_1 \in \left[0,1-\frac{\mu}{\tau}e^{-2}\right)$. Hence, for any $\frac{t}{\mu} \le \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$, there is no profitable deviation for any $s_1 \in \left[0,1-\frac{\mu}{\tau}e^{-2}\right)$.

Now consider deviations $s_1 \in \left(1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{n\tau}\right)$ when $s_2^* = 1$.

Intermediary 1's profit is given by

$$\tilde{\pi}_1(s_1) = \frac{\rho\mu^2}{\mu-(1-s_1)\tau}\left(\frac{1}{2t}\right)\left\{t + \mu\left[\ln\left(\frac{\mu}{(1-s_1)\tau}\right) - \frac{\mu}{\mu-(1-s_1)\tau} - 1 - \ln(n) + \frac{n}{n-1}\right]\right\}.$$

It can be shown that

$$\frac{\partial\pi_1}{\partial s_1} > 0 \iff \frac{t}{\mu} < \frac{\mu}{(1-s_1)\tau} + \frac{2\mu}{\mu-(1-s_1)\tau} - \ln\left[\frac{\mu}{(1-s_1)\tau}\right] - \frac{n}{n-1} + \ln(n) \equiv f(s_1).$$

where $f(s_1) > 0$ is increasing on $\left(1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{n\tau}\right)$.

Suppose $0 \le \frac{t}{\mu} \le e^2 + \frac{2}{e^2-1} - \frac{n}{n-1} + \ln(n) = f\left(1-\frac{\mu}{\tau}e^{-2}\right)$. Then $\frac{\partial\pi_1}{\partial s_1} > 0$ for all $s_1 \in \left(1-\frac{\mu}{\tau}e^{-2},\ 1-\frac{\mu}{n\tau}\right)$. Thus,

$$\tilde{\pi}_1(s_1) \le \tilde{\pi}_1\left(1-\frac{\mu}{n\tau}\right) = \frac{\rho\mu}{2}\left(\frac{n}{n-1}\right)\left(1-\frac{\mu}{t}\right)$$

for all $s_1 \in \left(1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{n\tau}\right)$. However, by choosing $s_1^* = 1$, intermediary 1 earns $\pi_1^* = \frac{\rho\mu}{2(1-1/n)}$, which exceeds $\tilde{\pi}_1\left(1-\frac{\mu}{n\tau}\right)$.

Suppose $e^2 + \frac{2}{e^2-1} - \frac{n}{n-1} + \ln(n) < \frac{t}{\mu} < \frac{n(e^2-1)}{n-e^2}\left[\ln(n) - \frac{n}{n-1} + \frac{1}{e^2-1}\right]$. Then $\frac{\partial\pi_1}{\partial s_1} < 0$ near $s_1 = 1 - \frac{\mu}{\tau}e^{-2}$. In this case, $\tilde{\pi}_1(s_1)$ is bounded by either $\tilde{\pi}_1\left(s_1 = 1-\frac{\mu}{\tau}e^{-2}\right)$ or $\tilde{\pi}_1\left(s_1 = 1-\frac{\mu}{n\tau}\right)$. We know from above that both of these values are exceeded by the profit π_1^*. Hence there is no profitable deviation $s_1 \in \left(1-\frac{\mu}{\tau}e^{-2}, 1-\frac{\mu}{n\tau}\right)$.

Finally consider any deviation $s_1 \in \left(1-\frac{\mu}{n\tau}, 1\right)$. This leads to profits given by

$$\tilde{\pi}_1(s_1) = \frac{\rho\mu}{2t}\left(\frac{n}{n-1}\right)[t - (1-s_1)n\tau],$$

which is obviously increasing in s_1. Therefore, choosing $s_1^* = 1$ gives intermediary 1 more profit than any in $s_1 \in \left(1 - \frac{\mu}{n\tau}, 1\right)$.■