

Collusion under Imperfect Monitoring with Asymmetric Firms*

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Abstract

We explore the effects of asymmetries in capacity constraints on collusion where market demand is uncertain and where firms must monitor the agreement through their privately observed sales and prices. Using this limited information, we show that all firms can always infer when at least one firm's sales are below some firm-specific "trigger level". This public information ensures that firms can detect deviations perfectly if fluctuations in market demand are sufficiently small. Otherwise, there is imperfect monitoring and punishment phases must occur on the equilibrium path. We find that asymmetries always hinder collusion. Yet, we also show that the competitive prices of asymmetric capacity distributions are actually higher than the collusive prices of less asymmetric capacity distributions, if the fluctuations in market demand are sufficiently large. We draw conclusions for merger policy.

JEL classification: D43, D82, K21, L12, L41

Key words: capacity constraints, mergers, collusion, imperfect monitoring

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1 Introduction

The recent collusion theory literature has developed a clear consensus that asymmetries hinder collusion. For example, this result is robust to whether asymmetries are in terms of firms' capacity constraints (see Compte *et al.*, 2002; Vasconcelos, 2005; and Bos and Harrington, 2010 and 2014) or the number of differentiated products that each firm sells (see Kühn, 2004). These papers in particular have been important for merger policy as they have highlighted which types of mergers can cause coordinated effects, that is, an increased likelihood or sustainability of tacit collusion post-merger. More specifically, with respect to capacity constraints, Compte *et al.* (2002) show that collusion is more difficult as the capacity of the largest firm is increased through a merger, and Vasconcelos (2005) finds that collusion is hindered when the largest firm is larger or when the smallest firm is smaller. Bos and Harrington (2010) show that increasing the capacity of medium-sized firms can facilitate collusion, if only a subset of firms in the market are involved in the collusion.¹

In practice, the degree to which firms can monitor each other's actions plays an important part in determining whether a merger causes coordinated effects. Yet, all of the papers above assume there is perfect observability of rivals' actions, so deviations from the collusive strategies will be detected immediately. In contrast, many mergers occur in markets in which there is the potential for secret price cuts. This may be the case, for example, in upstream business-to-business markets where transaction prices can be unrelated to posted prices. Consequently, it is inappropriate to consider the effects of such mergers in terms of collusion under perfect observability. Instead, they should be considered in the context of imperfect monitoring, where firms are uncertain over whether their rivals have followed their collusive strategies or not (see Green and Porter, 1984; Harrington and Skrzypacz, 2007 and 2011). However, while the models in this literature provide many interesting insights into the sustainability of collusion, it is difficult to draw implications for merger policy from them, because they analyse collusion with symmetric firms.

In this paper, we begin to fill this gap in the literature by exploring the effects of asymmetries in capacity constraints on collusion under imperfect monitoring. We achieve this by extending Compte *et al.* (2002) to a setting where there is demand uncertainty and where firms never directly observe their rivals' prices or sales. Thus, similar to the imperfect monitoring setting first discussed by Stigler (1964), each firm must monitor the collusive agreement using their

¹Fonseca and Normann (2008, 2012) also find that asymmetries in capacity constraints hinder collusion in laboratory experiments.

own privately observed sales. In this regard, our model is related to Tirole's (1988, p.262-264) model of private monitoring that captures the results of Green and Porter (1984) in a Bertrand framework (see also Campbell *et al.*, 2005, and Amelio and Biancini, 2010). Yet, unlike Tirole (1988), where there is a chance in each period that market demand will be zero, in our model market demand is drawn from an interval, where all possible states are positive. We use this model to investigate whether collusion is facilitated or hindered as capacity is reallocated among the firms to draw implications for merger policy.

Using information from their privately observed sales, we show that all firms can always infer when at least one firm's sales are below some firm-specific "trigger level". The trigger level for each firm is determined by the largest possible sales consistent with them or a rival being undercut on price. Thus, if all firms set a common price, then all firms' sales will exceed their respective trigger levels when the realisation of market demand is high, otherwise they can all fall below the trigger levels. Yet, if all firms do not set a common price, then at least one firm will receive sales below their trigger level. We restrict attention to equilibria in public strategies, where firms condition their play upon this public information, that is, whether all firms' sales are greater than their trigger levels or not.² Such strategies ensure monitoring is perfect if fluctuations in market demand are small, because firms will only ever receive sales below their trigger levels if they are undercut. However, collusive sales can also fall below the trigger levels, if fluctuations in market demand are large. Consequently, in contrast to Compte *et al.* (2002), there is uncertainty as to whether rivals have followed the collusive strategies or not, so punishment periods must occur on the equilibrium path to provide firms with the correct incentives to collude.

We find that asymmetries hinder collusion whether monitoring is perfect or imperfect. For instance, the critical discount factor is higher when the largest firm is larger or when the smallest firm is smaller. The reason for the former is that there is a greater incentive for the largest firm to deviate in a punishment period when it has more capacity, so the punishment must be weaker. The latter is due to the fact that deviations by the smallest firm are most difficult for rivals to detect, because each rival's resultant sales are most similar to its collusive sales. Thus, decreasing the size of the smallest firm makes monitoring more difficult. Another implication of this is that the optimal equilibrium profits are lower when the smallest firm is smaller. The reason is that punishment phases occur more often on the equilibrium path when the smallest firm has less

²In the main paper, we focus on symmetric public strategies, where firms follow identical strategies after every public history. In an appendix, we generate the same main results by solving the game following the approach of Tirole (1988). This does not rely on symmetric strategies in our setting.

capacity, since monitoring is more difficult. The capacities of the medium-sized firms do not affect the critical discount factor or the equilibrium profits.

After solving the model, we then use it to draw implications for merger policy. In particular, we analyse both the coordinated and unilateral effects of mergers in a unified framework. Unilateral effects arise if any firm is likely to have an individual incentive to raise prices post-merger. It is well understood that such effects are associated with asymmetric post-merger market structures and coordinated effects are associated with symmetric post-merger market structures (see Ivaldi *et al.*, 2003a and 2003b). In terms of the previous literature, these effects have been modelled independently of each other. For example, in the framework of Compte *et al.* (2002), either the monopoly price is sustainable, in which case only coordinated effects matter, or collusion is not sustainable at any price, so only unilateral effects matter. In contrast, our model allows for a more continuous treatment of such effects, because play alternates between phases of collusion and competition on the equilibrium path.

The conventional wisdom is that coordinated effects are more harmful to welfare than unilateral effects. The reason, as described by Röller and Mano (2006, p.22), is that “it is preferable that any coordination is by only a subset of firms (i.e. the merging parties) rather than all firms (tacitly)”. In other words, the fear is that firms will share the monopoly profits in every future period if collusion is sustainable, so only a merger to monopoly would be equally as bad in terms of unilateral effects. This logic also implies that a merger that disrupts collusion, by enhancing the market power of a single firm, should increase consumer surplus post-merger. In contrast, we show, as conjectured by Kühn (2001) and Motta *et al.* (2003), that this conventional wisdom is not always true under imperfect monitoring. This is due to the fact that firms will not be able to share the monopoly profits, because punishment phases occur on the equilibrium path. Consequently, a merger that facilitates collusion by distributing capacity symmetrically can be less harmful to welfare than one that creates a near monopoly. We demonstrate that the competitive prices of asymmetric capacity distributions are higher than the collusive prices of symmetric capacity distributions, if the fluctuations in market demand are sufficiently large.

Finally, our model is distinct from the previous literature that analyses collusion with capacity constraints and fluctuations in market demand. The main difference is that our focus is on mergers, which necessarily requires us to model asymmetries in markets with more than two firms. In contrast, the focus of this other literature is on pricing over the business cycle. For instance, Staiger and Wolak (1992) and Knittel and Lepore (2010) endogenise the choice of capacities in an infinitely repeated game. Despite analysing asymmetric games following the capacity choice

stage, they restrict attention to duopoly. Other differences are that there is perfect observability and market demand is known when prices are set. Under similar assumptions, Fabra (2006) analyses collusion with exogeneous capacity constraints but with symmetric firms.

The rest of the paper is organised as follows. Section 2 sets out the assumptions of the model and solves for the static Nash equilibrium. In section 3, we analyse the repeated game. We first show that there is some public information that firms can condition their play on, and find when monitoring is perfect or imperfect. Then we solve the game and discuss the comparative statics, drawing implications for merger policy. In section 4, we analyse an example to show that symmetric collusive capacity distributions can have substantially higher consumer surplus than asymmetric noncollusive capacity distributions. Section 5 explores the robustness of our results, and section 6 concludes. All proofs are relegated to appendix A. In appendix B, we solve the game following the approach of Tirole (1988) and show that this generates the same main results. This appendix is best read after section 3.2.

2 The Model

2.1 Basic assumptions

Consider a market in which a fixed number of $n \geq 2$ capacity-constrained firms compete on price to supply a homogeneous product over an infinite number of periods. Firms' costs are normalised to zero and they have a common discount factor, $\delta \in (0, 1)$. In any period t , firms set prices simultaneously where $\mathbf{p}_t = \{p_{it}, \mathbf{p}_{-it}\}$ is the vector of prices set in period t , p_{it} is the price of firm $i = \{1, \dots, n\}$ and \mathbf{p}_{-it} is the vector of prices of all of firm i 's rivals. Market demand consists of a mass of m_t (infinitesimally small) buyers, each of whom are willing to buy one unit provided the price does not exceed 1, without loss of generality. We assume that firms are uncertain of the level of market demand but they know that m_t is independently drawn from a distribution $G(m)$, with mean \hat{m} and density $g(m) > 0$ on the interval $[\underline{m}, \bar{m}]$.

Buyers are informed of prices, so they will want to buy from the cheapest firm. However, the maximum that firm i can supply in any period is k_i , where we let $k_n \geq k_{n-1} \geq \dots \geq k_1 > 0$, without loss of generality. We denote total capacity as $K \equiv \sum_i k_i$ and the maximum that firm i 's rivals can supply in each period as $K_{-i} \equiv \sum_{j \neq i} k_j$. In contrast to the buyers, firm i never observes firm j 's prices, $p_{j\tau}$, or sales, $s_{j\tau}$, $j \neq i$, for all $\tau \in \{0, \dots, t-1\}$. Thus, similar to Tirole (1988), our setting has the feature that all buyers are fully aware of prices, yet all firms are only

aware of their own prices. Such a setting is consistent with a market in which all buyers are willing to check the prices of every firm in each period to find discounts from posted prices, but actual transaction prices are never public information.³

2.2 Demand rationing and sales

Following the other papers in the literature (for example, Vasconcelos, 2005; and Bos and Harrington, 2010 and 2014), we make the common assumption that demand is allocated using the proportional rationing rule. This rule is as follows:

The proportional rationing rule

- Demand is allocated to the firm with the lowest price first. If this firm's capacity is exhausted, then demand is allocated to the firm with the second lowest price, and so on.
- If two or more firms set the same price and if their joint capacity suffices to supply the (residual) demand, then such firms each receive demand equal to its proportion of the joint capacity.

This rationing rule is commonly considered in the literature in terms of firms selecting how much of the market demand each supplies. Indeed, there are a number of cartels that have allocated demand in proportion to each member's capacity (see the examples in Vasconcelos, 2005, and Bos and Harrington, 2010). However, it is less appropriate to think of the rationing rule in this manner in our model, because selecting how to share the market demand is likely to require some knowledge of market demand and rivals' sales, which is not present in our setup. Instead, we have tacit collusion in mind where buyers randomly allocate themselves among the firms with tied prices and spare capacity. More specifically, suppose each buyer randomly selects such a firm with a probability equal to the firm's proportion of the joint capacity. It then follows from the law of large numbers that the residual demand is allocated according to this rationing rule.

We also place the following plausible yet potentially restrictive assumption on the capacity distribution:

Assumption 1. $\underline{m} \geq K_{-1}$.

This says that the joint capacity of the smallest firm's rivals should not exceed the minimum market demand. This is a necessary condition that ensures firm i 's sales in period t are nonneg-

³In contrast to Tirole (1988), our main results simply require that enough buyers are informed of prices to hold, if capacity constraints are binding.

ative, for all i and m_t , even if it is the highest-priced firm.⁴ Thus, denoting $\Omega(p_{it})$ as the set of firms that price below p_{it} and $p_t^{\max} \equiv \max\{\mathbf{p}_t\}$, the proportional rationing rule and Assumption 1 together imply that firm i 's sales in period t , $s_{it}(p_{it}, \mathbf{p}_{-it}; m_t)$, for any $p_{it} \leq 1$, are:

$$s_{it}(p_{it}, \mathbf{p}_{-it}; m_t) = \begin{cases} k_i & \text{if } p_{it} < p_t^{\max} \\ \min \left\{ \frac{k_i}{K - \sum_{j \in \Omega(p_{it})} k_j} \left(m_t - \sum_{j \in \Omega(p_{it})} k_j \right), k_i \right\} \geq 0 & \text{if } p_{it} = p_t^{\max} \end{cases} \quad (1)$$

This says that a firm will supply its proportion of the residual demand if it is the highest-priced firm in the market and if capacity is not exhausted, otherwise it will supply its full capacity. This implies that firm i 's expected per-period profit is $\pi_{it}(p_{it}, \mathbf{p}_{-it}) = p_{it} \int_{\underline{m}}^{\overline{m}} s_{it}(p_{it}, \mathbf{p}_{-it}; m) g(m) dm$, where we drop time subscripts if there is no ambiguity. Furthermore, we write $\pi_i(p)$ if $p_j = p$ for all j , such that:

$$\pi_i(p) = \begin{cases} pk_i & \text{if } K \leq \underline{m} \\ pk_i \left(\int_{\underline{m}}^K \frac{m}{K} g(m) dm + \int_K^{\overline{m}} g(m) dm \right) & \text{if } \underline{m} < K < \overline{m} \\ pk_i \frac{\overline{m}}{K} & \text{if } \overline{m} \leq K, \end{cases}$$

for all i . So, such profits are maximised for $p^m \equiv 1$.

To understand the generality of Assumption 1, note that it is not restrictive if all firms can only ever collectively supply as much as the minimum market demand, $\underline{m} \geq K$. Otherwise, for a given level of \underline{m} , there is a restriction on the size of the smallest firm in that it cannot be too small. We believe that this is not very restrictive in the context of coordinated effects. For example, using data from European merger decisions between 1990 and 2004, Davies *et al.* (2011) estimate that the European Commission would be expected to intervene due to concerns of tacit collusion, only if the smallest firm has a market share in excess of 30% post-merger.⁵ Translating this result into our setting by supposing market shares are proportional to capacity, Assumption 1 would then hold for such conditions if the minimum market demand is greater than 70% of the total capacity, $\underline{m} \geq 0.7K$. Moreover, the smallest firm's capacity can be no larger than for a symmetric duopoly, so a necessary (but not sufficient) condition for Assumption 1 to hold is that the minimum market demand must be greater than 50% of the total capacity, $\underline{m} \geq 0.5K$. Thus, it is clear that Assumption 1 comes with some loss of generality, but it is likely

⁴It also contrasts with Tirole's (1988) model, which requires the less realistic assumption that the minimum market demand is zero in some periods.

⁵This implies triopoly at most, and all but one of the mergers that raised concerns of coordinated effects over this period were for duopolies.

to hold for a large number of mergers that raise concerns of coordinated effects.⁶ We place no restriction on the level of the maximum market demand, \bar{m} .

2.3 Static Nash equilibrium

In this subsection, we analyse the stage game. Consistent with the standard Bertrand-Edgeworth setting, the static Nash equilibrium can be in pure strategies or mixed strategies. While the proof of the former is trivial, we extend the equilibrium analysis in Fonseca and Normann (2008) to our setting of demand uncertainty to solve for the latter. This is also equivalent to the equilibrium analysis of Gal-Or (1984) if firms are symmetric.

Lemma 1. *For any given $n \geq 2$ and $K_{-1} \leq \underline{m}$, there exists:*

- i) a unique pure strategy Nash equilibrium, such that $\pi_i^N = k_i$ for all i , if $\underline{m} \geq K$, and*
- ii) a mixed strategy Nash equilibrium, such that, for all i :*

$$\pi_i^N(k_i, k_n, \hat{m}) = \begin{cases} \frac{k_i}{k_n} \left(\int_{\underline{m}}^K (m - K_{-n}) g(m) dm + k_n \int_K^{\bar{m}} g(m) dm \right) & \text{if } \underline{m} < K < \bar{m} \\ \frac{k_i}{k_n} (\hat{m} - K_{-n}) & \text{if } \bar{m} \leq K. \end{cases} \quad (2)$$

Competition is not effective if the minimum market demand is above total capacity, $\underline{m} \geq K$, so firms set $p_i = 1$ and receive $\pi_i^N = k_i$ for all i . In contrast, if market demand can be below total capacity, firms are not guaranteed to supply their full capacity for every level of demand, so they have incentives to undercut each other. However, by charging $p_i = 1$, firm i can ensure that its expected per-period profit is at least:

$$\bar{\pi}_i \equiv \begin{cases} \int_{\underline{m}}^K (m - K_{-i}) g(m) dm + k_i \int_K^{\bar{m}} g(m) dm & \text{if } \underline{m} < K < \bar{m} \\ \hat{m} - K_{-i} & \text{if } \bar{m} \leq K. \end{cases} \quad (3)$$

This defines firm i 's minimax payoff. The intuition is that the firm with strictly the highest price expects to supply its full capacity if the realisation of market demand exceeds total capacity, but it expects to supply the residual demand otherwise. It follows from this that the largest firm will never set a price below $\underline{p} \equiv \bar{\pi}_n/k_n$ in an attempt to be the lowest-priced firm. This implies that the smaller firms $i < n$ can sell their full capacity with certainty by charging a price slightly below \underline{p} to obtain a profit of $k_i \bar{\pi}_n/k_n > \bar{\pi}_i$. Consequently, there exists a mixed strategy Nash equilibrium with profits given by (2), where Assumption 1 is sufficient to ensure that these are nonnegative for all i . The lower bound of the support is \underline{p} .

⁶Furthermore, Assumption 1 is actually sufficient for our main results, because we have generated similar results for duopoly when Assumption 1 is relaxed. These results are available from the authors upon request.

3 Monitoring with Asymmetries

In this section, we analyse the repeated game. We first show that there is some public information that firms can condition their play on, and find when monitoring is perfect or imperfect. We then solve the game, analyse the comparative statics, and draw implications for merger policy. Henceforth, we impose $\underline{m} < K$, as collusion is unnecessary otherwise from Lemma 1.

3.1 Information and monitoring

Under our assumptions, repetitions of the stage game generate private and public information histories. For instance, the private history of firm i in period t is the sequence of its past prices and sales, denoted $z_i^t \equiv (p_{i0}, s_{i0}; \dots; p_{it-1}, s_{it-1})$. In contrast, a public history is the sequence of information that is observed by all firms, regardless of their actions. In this subsection, we show that the fact that each firm observes its own sales implies that all firms will always know when at least one firm's sales are below some firm-specific "trigger level". As we discuss below, firms can then use public strategies in which they condition their play on this public information.

Formally, let $m^*(k_1, \bar{m}) \equiv \frac{K(\bar{m}-k_1)}{K-1}$ where firm i 's trigger level is $s_i^* \equiv \min \left\{ \frac{k_i}{K} m^*(k_1, \bar{m}), k_i \right\}$ for all i . As we show below, such trigger levels are determined by the largest possible sales firms $i > 1$ can make if all such firms set the same price and firm 1 undercuts. This then guarantees that at least one firm will always receive sales below their trigger level, if all firms do not set a common price. Now consider the history $h^t = (y_0, y_1, \dots, y_{t-1})$ where, for all $\tau = \{0, 1, \dots, t-1\}$:

$$y_\tau = \begin{cases} \bar{y} & \text{if } s_{i\tau}(p_{i\tau}, \mathbf{p}_{-i\tau}; m_\tau) > s_i^* \forall i \\ \underline{y} & \text{otherwise.} \end{cases}$$

This says that $y_\tau = \bar{y}$ if all firms' sales in period τ exceed their trigger levels, but $y_\tau = \underline{y}$ if at least one firm's sales does not.

We wish to establish that h^t is a public history. This requires that y_τ is common knowledge for all τ , for any z_i^t . Clearly, this is the case if the trigger levels are so high that all firms' sales can never exceed them for any prices, that is, $s_i^* = k_i$ so $y_\tau = \underline{y}$ for all τ . This occurs only if the maximum market demand is above the total capacity, $\bar{m} \geq K$, because then a firm is uncertain as to whether a rival has undercut it on price, even if the firm sells its full capacity. So consider $\bar{m} < K$, where it is possible for firms to receive sales above their trigger levels, since $s_i^* < k_i$. In this case, if all firms do not set a common price, then the sales of the firm(s) with the highest price in the market will never exceed their trigger levels. For instance, for any nonempty set of

rivals with a price below p^{\max} , $\Omega(p^{\max})$, the sales of firm i with $p_i = p^{\max} \leq 1$ are:

$$s_i = \frac{k_i}{K - \sum_{j \in \Omega(p^{\max})} k_j} \left(m_t - \sum_{j \in \Omega(p^{\max})} k_j \right) \leq \frac{k_i(\bar{m} - k_1)}{K-1} = s_i^* < k_i, \quad (4)$$

from (1). This guarantees that h^t is also a public history if $\bar{m} < K$ for the following reasons. If all firms set a common price $p \leq 1$, then the sales of all firms will exceed their respective trigger levels if the realisation of market demand is high, otherwise they can all fall below the trigger levels. Yet, as has just been demonstrated, if all firms do not set such a common price, then the sales of the firms that set the highest price will not exceed their trigger levels and all of their rivals will supply their full capacities.⁷ Any firm that supplies its full capacity can infer from this that at least one firm's sales are below its trigger level. The reason is that each firm knows, from (1), that it will supply its full capacity only if its price is below the highest in the market.⁸

This public information allows firms to make inferences about the behaviour of their rivals. In particular, each firm knows that all firms' sales will exceed their trigger levels, such that $y = \bar{y}$, only if $p_j = p \leq 1$ for all j and if $m > m^*(k_1, \bar{m})$; otherwise, at least one firm's sales will not exceed its trigger level, so $y = \underline{y}$. It follows from this that there is perfect monitoring of a strategy in which all firms set a common collusive price, if $\underline{m} > m^*(k_1, \bar{m})$. This is due to the fact that each firm would only receive sales below its trigger level, if it has been undercut. In contrast, there is imperfect monitoring of such an agreement, only if $\underline{m} \leq m^*(k_1, \bar{m})$. The reason can be understood by considering $\Pr(\underline{y}|p_i, \mathbf{p}_{-i})$ which denotes the probability of observing \underline{y} if firm i sets p_i and its rivals price according to \mathbf{p}_{-i} . For the case of $\underline{m} \leq m^*(k_1, \bar{m})$:

$$\Pr(\underline{y}|p_i, \mathbf{p}_{-i}) = \begin{cases} G(m^*(k_1, \bar{m})) = \int_{\underline{m}}^{\min\{m^*(k_1, \bar{m}), \bar{m}\}} g(m) dm \in [0, 1] & \text{if } p_j = p \ \forall j \\ 1 & \text{otherwise.} \end{cases} \quad (5)$$

This says that a firm's sales can be below their trigger level if the realisation of market demand is sufficiently low, even when firms set a common price. Thus, for such an outcome colluding firms face a non-trivial signal extraction problem: each firm does not know whether the realisation of market demand was unluckily low or whether at least one rival has undercut them.

⁷Likewise, if any firms' prices are above 1, then they will receive zero sales, which is below their trigger levels.

In this case, only the firms whose prices do not exceed 1 will supply their full capacities.

⁸Notice that if the trigger levels were below s_i^* for all i , then a firm that supplies its full capacity would be uncertain as to whether at least one rival has received sales below its trigger level. So, any such trigger levels would not generate a public history. In contrast, trigger levels above s_i^* for all i would also ensure that h^t is a public history. However, such trigger levels have the strange feature that firms can receive a bad signal \underline{y} , even when all firms know that they have set a common price. Consequently, such alternative trigger levels are inferior to s_i^* : they raise the critical discount factor and lower equilibrium profits compared to the main analysis.

Proposition 1 finds the conditions for perfect and imperfect monitoring in terms of the maximum market demand, holding the minimum market demand constant.

Proposition 1. *For any given $n \geq 2$, $K_{-1} \leq \underline{m} < K$, and $\delta \in (0, 1)$, there exists a unique level of market demand, $\underline{x}(k_1) \in (\underline{m}, K)$, such that monitoring is perfect if $\bar{m} < \underline{x}(k_1)$. Otherwise, there is imperfect monitoring.*

Monitoring is perfect if the fluctuations in market demand are sufficiently small, otherwise there is imperfect monitoring. The critical level is strictly increasing in the capacity of the smallest firm, k_1 . The reason is that deviations by the smallest firm are most difficult to detect, from (4). Furthermore, it follows from this logic that detecting deviations is less difficult when the smallest firm is larger. Consequently, if it is just possible for a firm to infer that the smallest firm has not deviated for a given level of \bar{m} , then it is also possible for the same level of \bar{m} if the smallest firm has more capacity. This implies that deviations can be detected perfectly for a wider range of fluctuations in market demand if the smallest firm is larger.

Finally, we have so far considered the public information that firms can infer from their privately observed sales. Before moving on, we should discuss two possible scenarios in which a firm's sales can provide it with private information that is not common knowledge among all firms. In either case though, it should be noted that any such private information is not payoff relevant if rivals follow public strategies. Thus, it will not be possible for a firm to use its private information to gain by deviating from an equilibrium in public strategies. The first case is when a firm knows for sure that it has been undercut. This occurs if firm i 's sales are inconsistent with all firms setting a common price, $s_i < \frac{k_i}{K} \bar{m}$ for some i . Such information is not common knowledge if monitoring is imperfect, because the deviants $j \neq i$ would be unaware of the specific levels of its rivals' sales: they simply know that at least one rival's sales are below its trigger level. The second case is when the smallest firm knows for sure that all firms have set a common price, but its rivals $i > 1$ are uncertain as to whether the smallest firm has undercut them. This may occur only if firm 1 is strictly the smallest firm and if fluctuations in market demand are not large, such that $\bar{m} < K$. In such a case, the highest possible sales of the smallest firm if it is undercut are $\frac{k_1}{K_{-2}} (\bar{m} - k_2) < s_1^*$. Thus, if the smallest firm's sales are below its trigger level yet above $\frac{k_1}{K_{-2}} (\bar{m} - k_2)$, then it knows for sure that all firms have set a common price. Nevertheless, the fact that its sales are below its trigger level will inform the smallest firm that its rivals' sales are also below their trigger levels.

3.2 Optimal symmetric equilibrium payoffs

We now solve the repeated game restricting attention in this subsection to symmetric perfect public equilibria (SPPE) in pure strategies (see Fudenberg and Tirole, 1994, p.187-191). Such equilibria are sequential equilibria in public strategies, in which firms condition their play only on the public history. Such strategies are strongly symmetric in the sense that each firm uses an identical strategy after every public history. This implies that firms are required by the strategy to set common prices in future periods, even if they have not set common prices in the past. Thus, if such strategies prescribe a pricing path $\{p_t\}_{t=1}^{\infty}$ for some history h^t , then firm i 's expected (normalised) profits are:

$$(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(p_t) \equiv k_i V(h^t),$$

where $V(h^t) \equiv \frac{(1-\delta)}{K} \sum_{t=1}^{\infty} \delta^{t-1} \sum_i \pi_i(p_t)$ is the expected (normalised) profits per unit of capacity. Consequently, the expected future punishments and rewards for each firm depend upon how much capacity it has. Furthermore, our strong symmetry assumption restricts attention to symmetric subgame perfect Nash equilibria (SPNE) if monitoring is perfect, because then the set of perfect public equilibria (PPE) coincide with the set of SPNE. We say that collusion is not sustainable if no such equilibria exist.

We are interested in finding an optimal SPPE that supports the highest SPPE payoffs. Thus, following Abreu *et al.* (1986, 1990), we note that any public strategy profile can be transformed into firm i 's profits of the stage game and a continuation payoff function. The continuation payoff function, $w_i(y)$, describes the expected (normalised) future profit of firm i depending upon the realisation of y from the stage game, and the strong symmetry assumption implies it is of the form $k_i V(h^t)$. We define $\mathbb{E}[w_i(y) | p_i, \mathbf{p}_{-i}]$ as the expected (normalised) continuation payoff if firm i sets p_i and expects its rivals to price according to \mathbf{p}_{-i} . Thus, firm i 's expected (normalised) profit from a symmetric public strategy is $(1 - \delta) \pi_i(p) + \delta \mathbb{E}[w_i(y) | p]$ for all i . Let $E(\delta, \bar{m})$ be the (possibly empty) set of SPPE payoffs for a given δ , and let \bar{V} and \underline{V} be the highest and lowest SPPE payoffs per unit of capacity, when the set is nonempty.

An optimal SPPE chooses a profile p and a function $w_i(y)$ to maximise a firm's expected profits subject to the constraints that all the continuation payoffs correspond to SPPE profiles, $w_i(y) \in E(\delta, \bar{m})$ for all i , and that, for all $p_i \neq p$,

$$(1 - \delta) \pi_i(p) + \delta \mathbb{E}[w_i(y) | p] \geq (1 - \delta) \pi_i(p_i, p) + \delta \mathbb{E}[w_i(y) | p_i, p] \quad \forall i. \quad (6)$$

This says that no firm must be able to gain by a (one-stage) deviation from the symmetric

strategy. Furthermore, making the common assumption that there is some publically observable randomisation device ensures that a nonempty set of SPPE payoffs is convex. Thus, such a set can be represented by the interval $[\underline{V}, \overline{V}]$, where firm i 's SPPE payoff is $k_i V$ for any given $V \in [\underline{V}, \overline{V}]$, for all i . It then follows from the so-called bang-bang property that the highest SPPE payoff for firm i can be characterised by restricting attention to SPPEs that threaten to switch to either $k_i \overline{V}$ or $k_i \underline{V}$.

More specifically, we assume that, after observing y_t , the firms observe the realisation of the public randomisation device, and it has the following implications for their behaviour. If firms observe \bar{y} in period t , then period $t+1$ is a ‘‘punishment period’’ with probability $\alpha \in [0, 1]$ and it is a ‘‘collusive period’’ otherwise, where the continuation payoffs are $k_i \underline{V}$ in punishment periods and $k_i \overline{V}$ in collusive periods from the bang-bang property. Yet, if firms observe \underline{y} in period t , then period $t+1$ is a punishment period with probability $\beta \in [0, 1]$ and it is a collusive period otherwise. Thus, letting $\theta \equiv (1 - \Pr(\underline{y}|p)) \alpha + \Pr(\underline{y}|p) \beta$, it follows from the above and (5) that firm i 's expected continuation payoff if it expects its rivals to set p is:

$$\mathbb{E}[w_i(y) | p_i, p] = \begin{cases} k_i [\theta \underline{V} + (1 - \theta) \overline{V}] & \text{if } p_i = p \\ k_i [\beta \underline{V} + (1 - \beta) \overline{V}] & \text{if } p_i \neq p. \end{cases}$$

Notice that $\theta = \beta$ for any $\bar{m} \geq K$, as then $\Pr(\underline{y}|p) = 1$ from (5). This implies $E(\delta, \bar{m})$ is an empty set for any δ if $\bar{m} \geq K$, because (6) does not ever hold. Consequently, we henceforth focus on the case of $\underline{m} \leq \bar{m} < K$.

Thus, we can characterise the highest and the lowest SPPE payoffs, and hence the set of SPPE payoffs, by solving the following constrained optimisation problem:

$$\overline{V} = \max_{\alpha, \beta, p^c, p^p} (1 - \delta) \frac{\hat{m}}{K} p^c + \delta [\theta \underline{V} + (1 - \theta) \overline{V}]$$

subject to:

$$\overline{V} = (1 - \delta) \frac{\hat{m}}{K} p^c + \delta [\theta \underline{V} + (1 - \theta) \overline{V}]$$

$$\underline{V} = (1 - \delta) \frac{\hat{m}}{K} p^p + \delta [\theta \underline{V} + (1 - \theta) \overline{V}]$$

$$k_i \overline{V} \geq (1 - \delta) \pi_i(p_i, p^c) + \delta k_i [\beta \underline{V} + (1 - \beta) \overline{V}] \quad \forall p_i \neq p^c, \forall i \quad (7)$$

$$k_i \underline{V} \geq (1 - \delta) \pi_i(p_i, p^p) + \delta k_i [\beta \underline{V} + (1 - \beta) \overline{V}] \quad \forall p_i \neq p^p, \forall i \quad (8)$$

$$0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, \underline{p} = \bar{\pi}_n / k_n < p^c \leq 1, p^p < p^c$$

This constrained optimisation problem solves for both the highest and the lowest SPPE payoffs in one step, because a higher \overline{V} will automatically allow for a lower \underline{V} , and vice versa. Regarding

the constraints, the first two are just identities that follow from the continuation payoffs described above, whereas (7) and (8) are the collusive period and punishment period incentive compatibility constraints (ICCs), respectively. The remaining constraints are also natural. Without loss of generality, we let the collusive price be $p^c > \underline{p}$, as this is a necessary (but not sufficient) condition for $k_i \bar{V}$ to be greater than the static Nash equilibrium profits. We do not require that the punishment price, p^p , is nonnegative to allow for the case where it is below marginal cost (which has been normalised to zero).⁹

To simplify our constrained optimisation problem, we consider which firms have the greatest incentives to deviate in collusive periods and in punishment periods to restrict attention to the ICCs that bind. To begin, notice that firm i 's optimal deviation profits from any given p , are:

$$\pi_i(p_i^*, p) \equiv \begin{cases} k_i p & \text{if } p > \bar{\pi}_i/k_i \\ \bar{\pi}_i & \text{if } p \leq \bar{\pi}_i/k_i, \end{cases} \quad (9)$$

where $\bar{\pi}_i = \hat{m} - K_{-i}$ for any $\bar{m} < K$ from (3). This says that it is optimal for firm i to deviate by marginally undercutting a common price if such a price is sufficiently high, otherwise it should supply the residual demand at the monopoly price. Thus, in a collusive period where $p^c \in (\underline{p}, 1]$, it follows that firm i 's optimal deviation profits are $\pi_i(p_i^*, p^c) = k_i p^c$. Substituting this into (7) clearly shows that the k_i 's cancel. This implies that if the collusive period ICC holds for firm i , then it also satisfied for all other firms $j \neq i$. In contrast, in a punishment period, the punishment price p^p could be above or below \underline{p} . Consequently, the incentives to deviate may differ for each firm. Nevertheless, regardless of the level of p^p , a necessary and sufficient condition for the punishment period ICC to be satisfied for all firms is that it holds for the largest firm. Thus, we only need consider the collusive period ICC for some firm i and the punishment period ICC for firm n .

Proposition 2 solves for the highest and lowest SPPE payoffs given there is imperfect monitoring. We refer to this as collusion under imperfect monitoring.

⁹This is an innocuous assumption. We could generate the same results, with $p^p \geq 0$, if marginal costs were sufficiently high. Alternatively, we could assume a separate randomisation device for punishment periods, with different probabilities compared to the collusive period randomisation device. This would ensure the firms have more instruments than just α , β , p^p and p^c . Furthermore, we actually generate the same results in appendix B, with nonnegative prices in the punishment phase and without the need of a randomisation device. However, if $p^p \geq 0$ is imposed, then the highest SPPE payoffs remain unchanged, but the critical discount factor is robust only if \hat{m} is sufficiently large (see the proof of Proposition 2 for more details). The critical discount factor is higher otherwise.

Proposition 2. For any given $n \geq 2$ and $K_{-1} \leq \underline{m} < K$, there exists a unique $\bar{x}(k_1, k_n) \in (\underline{x}(k_1), K)$, that solves $G(m^*(k_1, \bar{x}(k_1, k_n))) = 1 - \frac{k_n}{K} < 1$, such that the highest and lowest SPPE payoffs for firm i are:

$$k_i \bar{V} = \frac{k_i}{K} \left(\frac{\hat{m} - G(m^*(k_1, \bar{m}))K}{1 - G(m^*(k_1, \bar{m}))} \right) \in \left(\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \hat{m} \right) \quad \forall i,$$

$$k_i \underline{V} = k_i \bar{V} - (1 - \delta) \frac{k_i}{k_n} (K - \hat{m}) \in (\pi_i^N(k_i, k_n, \hat{m}), k_i \bar{V}) \quad \forall i,$$

if and only if $\delta \geq \delta^*(k_1, k_n) \equiv \frac{1}{1 - G(m^*(k_1, \bar{m}))} \frac{k_n}{K} \in (\frac{k_n}{K}, 1)$, for any $\underline{x}(k_1) \leq \bar{m} < \bar{x}(k_1, k_n)$. Otherwise, collusion is not sustainable.

Firms set the monopoly price in collusive periods with $\alpha^* = 0$ and $\beta^* > 0$ (the stars indicate optimal values), where β^* is set at the level where the collusive period ICC is binding with no slack. This implies that punishment periods occur on the equilibrium path when there is imperfect monitoring, so the equilibrium payoffs are below the monopoly level. The necessary conditions that ensure this is an optimal SPPE are found by noting that β^* must be less than or equal to 1. A lower price in punishment periods reduces β^* , so p^p is set low enough so that the punishment period ICC for the largest firm is binding with no slack. Furthermore, the critical discount factor must be less than 1, so that firms can be sufficiently patient. This in turn requires that there is a sufficiently low probability that firms' sales will be below their trigger levels when they set a common price. So, $\bar{x}(k_1, k_n)$ is implicitly defined as the level of the maximum market demand that sets the critical discount factor to 1.

Next, turning our attention to the case of perfect monitoring, we use the fact that the set of SPPE coincide with the set of symmetric SPNE when monitoring is perfect. Consequently, we can easily generate this set under perfect monitoring by setting $G(m^*(k_1, \bar{m})) = 0$ in the above. Thus, the set of symmetric SPNE payoffs is summarised by the following corollary. We refer to this as collusion under perfect monitoring.

Corollary 1. For any given $n \geq 2$ and $K_{-1} \leq \underline{m} < \bar{m} \leq \underline{x}(k_1)$, the highest and lowest SPNE payoffs for firm i are:

$$k_i \bar{V} = \frac{k_i}{K} \hat{m} > \pi_i^N(k_i, k_n, \hat{m}) \quad \forall i,$$

$$k_i \underline{V} = k_i \bar{V} - (1 - \delta) \frac{k_i}{k_n} (K - \hat{m}) \in [\pi_i^N(k_i, k_n, \hat{m}), k_i \bar{V}] \quad \forall i,$$

if and only if $\delta \geq \underline{\delta}^*(k_n) \equiv \frac{k_n}{K} \in (0, \frac{m}{K}]$. Otherwise, collusion is not sustainable.

The firms divide the monopoly profits between them if they are sufficiently patient. The critical discount factor is the same as the lowest critical discount factor in Compte *et al.* (2002). It also coincides with the lowest possible discount factor that sustains collusion given the proportional rationing rule. The reason is that, as showed by Lambson (1994), the optimal punishments under the proportional rationing rule are such that the largest firm receives the stream of profits from its minimax strategy. In our setting, the per-period minimax payoff of the largest firm is equivalent to its static Nash equilibrium profits. Moreover, the lowest possible SPNE payoff $k_i \underline{V}$ is also the static Nash equilibrium profits. Thus, it is not possible to lower the critical discount factor below this level, given the proportional rationing rule.

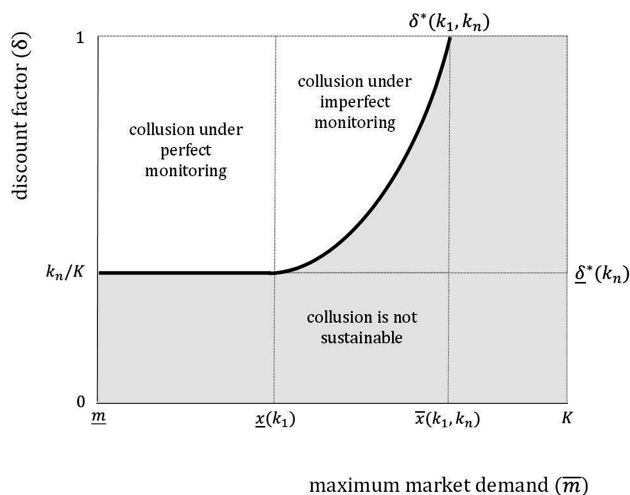


Figure 1: parameter space of collusion

These results are brought together in Figure 1. It highlights that the critical discount factor under imperfect monitoring, $\delta^*(k_1, k_n)$, converges to the critical level under perfect monitoring, $\underline{\delta}^*(k_n)$, at $\bar{m} = \underline{x}(k_1)$, but it is strictly above $\underline{\delta}^*(k_n)$ for any higher maximum market demand. To understand how the equilibrium profits change in Figure 1, note that the highest equilibrium payoffs, $k_i \bar{V}$, are independent of δ for any \bar{m} and that, assuming a mean-preserving spread, they are strictly decreasing in \bar{m} between $\underline{x}(k_1)$ and $\bar{x}(k_1, k_n)$ for a given δ . The latter also implies that the lowest equilibrium payoffs, $k_i \underline{V}$, are strictly decreasing in \bar{m} over the same range. Furthermore, the lowest equilibrium profits equal the static Nash equilibrium profits when evaluated at the critical discount factor, and they increase towards $k_i \bar{V}$ as δ tends to

1 for any given \bar{m} . This implies that $k_i \bar{V} = k_i \underline{V} = \pi_i^N(k_i, k_n, \hat{m})$ at $\bar{m} = \bar{x}(k_1, k_n)$ where $\delta^*(k_1, k_n) = 1$. Before moving on, the reader may wish to check appendix B, where we generate the same main results following an alternative approach.

3.3 Comparative statics

We want to analyse the effects of mergers in our setting. Before doing so, it is helpful to consider changes in the capacity distribution, when the number of firms and the total capacity are held constant. This will first provide a clear understanding of how the capacity distribution affects collusion. We discuss the effects of mergers in the following subsection. Under these assumptions, any such changes in the capacity of a given firm will require capacity to be reallocated from a rival. For example, increasing the size of the smallest firm in a duopoly implies that the capacity of the largest firm decreases. In general, when the capacity of firm j changes by a small amount, other things equal, the capacities of the other firms will have to change to the extent that $\frac{\partial k_i}{\partial k_j} \in [-1, 0]$ for all $i \neq j$, where $\sum_{i \neq j} \frac{\partial k_i}{\partial k_j} = -1$. In what follows, we show that only changes to the capacity of the smallest firm or the largest firm affect the equilibrium analysis.

Proposition 3 analyses the effects of reallocating capacity among the firms on the critical discount factor.

Proposition 3. *For any given $n \geq 2$ and $K_{-1} \leq \underline{m} < K$,*

- i) if $\underline{m} \leq \bar{m} < \underline{x}(k_1)$, then $\underline{\delta}^*(k_n)$ is strictly increasing in the capacity of the largest firm, k_n ,*
- ii) if $\underline{x}(k_1) \leq \bar{m} < \bar{x}(k_1, k_n)$, then $\delta^*(k_1, k_n)$ is strictly increasing in the capacity of the largest firm, k_n , and strictly decreasing in the capacity of the smallest, k_1 .*

Consistent with Compte *et al.* (2002), increasing the size of the largest firm hinders collusion. The reason is that the punishment is weaker when the largest firm is larger, as this ensures that the largest firm has no incentive to deviate in a punishment period. Consequently, the collusive phase ICC is tighter than before, so the critical discount factor rises. In contrast to Compte *et al.* (2002), increasing the size the smallest firm facilitates collusion. This is due to the fact that firms can monitor an agreement to set a common price through public information more successfully when the smallest firm is larger. This does not affect the critical discount factor under perfect monitoring but, as we saw in section 2.1, it does imply that monitoring is perfect for a wider range of fluctuations in market demand. Under imperfect monitoring, it is less likely that a collusive period will switch to a punishment phase on the equilibrium path. Consequently,

the collusive period ICC has more slack when the smallest firm is larger, so the critical discount factor falls.¹⁰

Next, we analyse the effects of reallocating capacity among the firms on the highest equilibrium payoffs. For convenience, we transform such payoffs to an average price and compare it to the average static Nash equilibrium price, given by $\hat{p}^N(k_n, \hat{m}) \equiv \frac{K}{\hat{m}} \frac{(\hat{m} - K_{-n})}{k_n}$ for all $\bar{m} < K$. The average price of the highest equilibrium payoffs under perfect monitoring is independent of the capacity distribution, since firms set p^m in each period if they are sufficiently patient. So, Proposition 4 investigates the effect of reallocating capacity on the average price associated with the highest SPPE payoffs under imperfect monitoring. We refer to this as the best average price, and this is given by $\hat{p}^c(k_1, \bar{m}) \equiv \frac{K}{\bar{m}} \bar{V}$ in expectation.

Proposition 4. *For any given $n \geq 2$, $K_{-1} \leq \underline{m} < \underline{x}(k_1) < \bar{m} < \bar{x}(k_1, k_n)$ and $\delta \geq \delta^*(k_1, k_n)$, the best average price $\hat{p}^c(k_1, \bar{m})$ satisfies $\hat{p}^N(k_n, \hat{m}) < \hat{p}^c(k_1, \bar{m}) < p^m$ and it is strictly increasing in the capacity of the smallest firm, k_1 .*

The best average price is increasing in the capacity of the smallest firm for two reasons. First, as the capacity of the smallest firm increases, it is less likely that firms' sales will be below their trigger levels when they set a common price. Thus, profits rise on the equilibrium path, other things equal, because collusive periods are less likely to switch to punishment periods than before. Second, such an increase in profits also introduces slack into the collusive phase ICC, so β^* falls to ensure that it is binding with no slack. Both effects imply that firms expect there to be more collusive periods on the equilibrium path than when the smallest firm has less capacity, so the best average price rises.

Surprisingly, the best average price is independent of the capacity of the largest firm. This is due to the fact that there are two effects that perfectly offset each other. The first effect is that an increase in the capacity of the largest firm raises profits on the equilibrium path, other things equal. The reason is that the punishment is weaker than before to ensure that the largest firm will not deviate in any punishment phase. However, this also tightens the collusive phase ICC, so the second effect is that β^* must increase to ensure that it is binding with no slack. This second effect cancels out the first, implying the size of the largest firm has no effect on the best average price.

¹⁰Both results are consistent with the findings of Vasconcelos (2005). The underlying incentives for his results are very different to ours though, as they rely on capacities affecting marginal costs in a setting of perfect observability.

It follows from the above analysis that asymmetries hinder collusion under perfect and imperfect monitoring. In summary, Proposition 3 implies that the parameter space of collusion is greatest when firms' capacities are symmetric, because the punishment is harshest when the largest firm is as small as possible, and since monitoring is most successful when the smallest firm is as large as possible. The latter also implies that the best average price is also higher when firms are symmetric from Proposition 4. Furthermore, since the best average price is independent of the size of the largest firm, it follows that best average price is highest for a symmetric duopoly and that, for example, it would be higher for a symmetric triopoly than an asymmetric duopoly with $k_1 < K/3$.

Despite the fact that asymmetries hinder collusion, Proposition 5 next shows that the competitive prices of asymmetric capacity distributions will be higher than the collusive prices of less asymmetric capacity distributions, if fluctuations in market demand are sufficiently large. To prove this result, we compare the best average price of one distribution, (k_1, k_n) , to the static Nash equilibrium average price of another, denoted (k'_1, k'_n) .

Proposition 5. *For any given $n \geq 2$ and $K_{-1} \leq \underline{m} < K$, there exists a unique $\bar{x}(k_1, k'_n) \in (\underline{x}(k_1), \bar{x}(k_1, k_n))$ if $k'_n > k_n$, that solves $G(m^*(k_1, \bar{x}(k_1, k'_n))) = 1 - \frac{k'_n}{K} < 1$, such that the static Nash equilibrium average price of (k'_1, k'_n) is greater than the best average price of (k_1, k_n) , $\hat{p}^N(k'_n, \hat{m}) > \hat{p}^c(k_1, \bar{m})$, if $\bar{x}(k_1, k'_n) < \bar{m} < \bar{x}(k_1, k_n)$ for any $\delta \geq \delta^*(k_1, k_n)$.*

The intuition is that an increase in the maximum market demand raises the likelihood that firms' sales will be below their trigger levels when firms set a common price. Thus, punishment periods are expected to occur more often than before on the equilibrium path. As a result, the best average price of (k_1, k_n) falls towards its corresponding static Nash equilibrium average price as the maximum market demand increases towards the critical level $\bar{x}(k_1, k_n)$. Yet, the average static Nash equilibrium price is strictly increasing in the capacity of the largest firm, k_n . Consequently, if the largest firm of an alternative distribution (k'_1, k'_n) has more capacity than the original, $k'_n > k_n$ (so $k'_1 \leq k_1$), then (k'_1, k'_n) will have a higher average static Nash equilibrium price than the best average price of (k_1, k_n) , $\hat{p}^N(k'_n, \hat{m}) > \hat{p}^c(k_1, \bar{m})$, if the maximum market demand is sufficiently close to $\bar{x}(k_1, k_n)$. The critical level of the maximum market demand above which this is true, $\bar{x}(k_1, k'_n)$, is the point at which $\hat{p}^c(k_1, \bar{m}) = \hat{p}^N(k'_n, \hat{m})$ for all $\delta \geq \delta^*(k_1, k_n)$, or expressed differently, the point where $k_i \bar{V} = \pi_i^N(k_i, k'_n, \hat{m})$. Furthermore, the condition that the maximum market demand exceeds this level guarantees that collusion is not sustainable for

(k'_1, k'_n) , since it contradicts the necessary condition in Proposition 2 that the maximum market demand is below $\bar{x}(k'_1, k'_n) \leq \bar{x}(k_1, k_n)$.

3.4 Implications for mergers

We now use our equilibrium analysis to draw implications for merger policy. In particular, we are interested in comparing the unilateral and coordinated effects in our framework. Such effects have been considered independently of each other in the previous literature. For instance, Compte *et al.* (2002) and Vasconcelos (2005) focus solely on the effects of mergers on the critical discount factor, because firms can share the monopoly profits if they are sufficiently patient. Bos and Harrington (2010) analyse the coordinated effects of mergers on the price of a cartel that does not encompass all firms in the market. They find that mergers that raise the capacity controlled by the cartel can increase the cartel price towards the monopoly level. However, in contrast to our model, they restrict attention to capacity distributions for which there is a unique pure strategy static Nash equilibrium price equal to marginal cost, so unilateral effects are not an issue. Thus, such papers are consistent with the conventional wisdom that collusive post-merger outcomes are worse than non-collusive outcomes. In our setting, collusion under imperfect monitoring does not enable firms to share the monopoly profits in every period. As a result, the conventional wisdom will not hold, if competition in the noncollusive outcome is weak and hence prices are high. Below we explore for which mergers the conventional wisdom does not hold.

The following analysis differs to the earlier comparative statics in that a merger will reduce the numbers of firms and that a merger can increase both the size of the smallest and largest firm at the same time. We also consider the firms' incentives to merge. Following the terminology of Farrell and Shapiro (1990), we henceforth refer to the merging firms as insiders and those not involved in the merger as outsiders. We say that a merger is privately optimal if the sum of insiders' profits post-merger is *strictly* greater than the sum of their profits pre-merger. Finally, with respect to welfare, we focus on the effects of mergers on consumer surplus, as this is commonly perceived to be the main objective of merger control (see Lyons, 2002).¹¹ Figure 2 depicts the effects of two mergers that change the equilibrium analysis either by only increasing the size of the smallest firm or by only increasing the size of the largest firm. We discuss each in turn. A merger that increases the size of both the smallest and the largest firm will have a mix of the following effects, and this issue is analysed more in the next section. All other mergers will not affect the equilibrium analysis.

¹¹Moreover, the expected total welfare is independent of the capacity distribution.

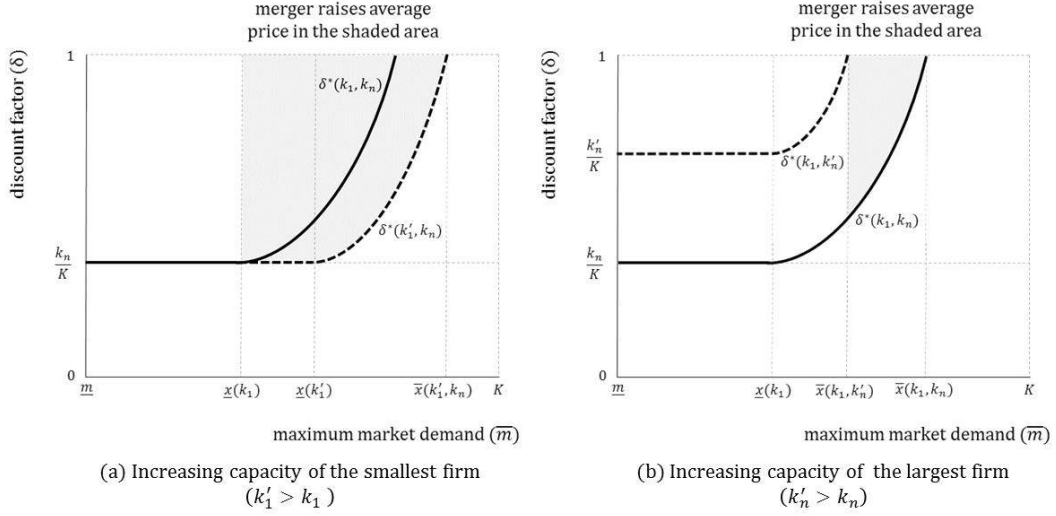


Figure 2: The effects of mergers

A merger that increases the size of the smallest firm will facilitate collusion. It follows from Proposition 3 that the parameter space of collusion will expand and Proposition 4 implies that the average price may also rise post-merger. More specifically, and as usual, the price will rise if collusion is not sustainable pre-merger but it is post-merger. Yet, in contrast to models where firms share the monopoly profits when colluding, such a merger will also raise the best average price, if there is collusion under imperfect monitoring pre-merger. Thus, the complete parameter space for which such a merger raises the average price is illustrated in the shaded area of Figure 2(a).¹² Any such merger that raises the average price is privately optimal and it also strictly increases the profits of the outsiders. This follows since the present discounted value of profits, given an average price \hat{p} , is $\sum_{i \in M} \frac{k_i}{K} \hat{m} \left(\frac{\hat{p}}{1-\delta} \right)$ for any subset of firms M , so this is strictly higher post-merger if the average price is higher. As a consequence, such a merger will also lower consumer surplus, since the expected consumer surplus per unit is $1 - \hat{p}$. Thus, consistent with the conventional wisdom, any collusive post-merger outcome that has been facilitated by an increase in the size of the smallest firm is worse than the pre-merger outcome.¹³

¹²The average price rises to $p^m = 1$ if there is collusion under perfect monitoring post-merger. It is below this level if there is collusion under imperfect monitoring. The merger has no effect on the average price if there is collusion under perfect monitoring pre- and post-merger.

¹³It follows from this that larger firms $i > 1$ can actually increase their profits by divesting capacity to the smallest firm, so that monitoring is easier. Such divestments are not unheard of in actual merger cases, because

A merger that increases the size of the largest firm will hinder collusion. Proposition 3 implies that the parameter space of collusion will be reduced. Nevertheless, in contrast to the conventional wisdom, it follows from Proposition 5 that such a merger may not actually decrease prices, if there is collusion under imperfect monitoring pre-merger but collusion is unsustainable post-merger. As illustrated in the shaded area of Figure 2(b), the average price is actually higher post-merger, if fluctuations in market demand are sufficiently large. In fact, our model suggests that it is only in the insiders' interests to propose a merger that destabilises collusion, if the average price rises post-merger. This follows since such a merger is privately optimal for any set of firms M if $\sum_{i \in M} \frac{k_i}{K} \widehat{m} \left(\frac{\widehat{p}^N(k'_n, \widehat{m})}{1-\delta} \right) > \sum_{i \in M} \frac{k_i}{K} \widehat{m} \left(\frac{\widehat{p}^c(k_1, \overline{m})}{1-\delta} \right)$. Consequently, the condition that guarantees the insiders' profits increase post-merger also ensures that the average price rises post-merger. Moreover, the same condition also guarantees that such a merger increases the profits of the outsiders and lowers consumer surplus.¹⁴

Thus, this suggests that the conventional wisdom does not always hold if a merger destabilises collusion under imperfect monitoring by increasing the size of the largest firm. The same conclusion applies if, in contrast to comparing pre- and post market structures as we have above, the comparison is instead between two possible merger outcomes, where one is asymmetric and noncollusive while the other is a less asymmetric and collusive. For example, this could arise if the merging parties offered to divest capacity to remedy concerns of coordinated effects. In either case, when a competition agency must decide between such market structures, it is important that there is consideration of the likelihood to which price wars will occur over time for the collusive distribution, and this should be compared against the effect of lessing competition through unilateral effects. Our models suggests that prices will be lower for the noncollusive asymmetric distribution if demand fluctuations are small, otherwise the symmetric collusive distribution has lower prices.

4 An Example

We complement our general results by analysing an example to show that symmetric collusive capacity distributions can have substantially lower average prices than asymmetric noncollusive capacity distributions. In our example, we suppose that total capacity is $K = 100$ and that this

there are examples where insiders' post-merger capacity or market shares fall as a result of a divestment remedy (see Compte *et al.*, 2002; and Davies and Olczak, 2010).

¹⁴Clearly, such a merger would also increase the average price post-merger, if collusion is not sustainable pre- or post-merger. This is not illustrated in Figure 2 to highlight the most interesting case.

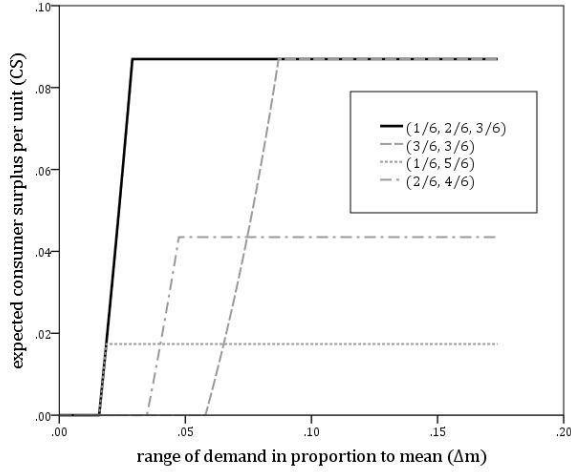


Figure 3: $G(m) = \frac{m-\underline{m}}{\bar{m}-\underline{m}}$, $\hat{m} = 92$, $K_{-1} \leq \frac{5}{6}(100) < 100 = K$, and $\delta \rightarrow 1$

is divisible into 6 equal sized parts. There is an asymmetric triopoly pre-merger, denoted $(1/6, 2/6, 3/6)$, where firm 1 has $1/6$ of this capacity, firm 2 has $2/6$ and firm 3 has $3/6$. We then consider three alternative merger outcomes: a symmetric duopoly, $(3/6, 3/6)$, resulting from a merger between firms 1 and 2; an asymmetric duopoly, $(2/6, 4/6)$, created by a merger between firms 1 and 3; and a very asymmetric duopoly $(1/6, 5/6)$, resulting from a merger between firms 2 and 3. An alternative way to consider these outcomes is that, for a given merger, the other duopoly outcomes arise from firms divesting capacities as remedies for anti-competitive effects.¹⁵

We analyse the effects of such mergers on the expected consumer surplus per unit of the most profitable equilibrium, denoted $CS(\hat{p}^*) \equiv 1 - \hat{p}^*$. The preceding analysis implies that \hat{p}^* is the static Nash equilibrium average price if collusion is not sustainable, otherwise it is the best average price or the monopoly price. Figure 3 plots $CS(\hat{p}^*)$ as a function of $\Delta m \equiv \frac{\bar{m}-\underline{m}}{\hat{m}}$ for the various scenarios assuming demand is drawn from a uniform distribution. Parameter values are chosen such that $\hat{m} = 92$ for all Δm and that $K_{-1} \leq \frac{5}{6}(100) \leq \underline{m} \leq \bar{m} \leq K = 100$, so Assumption 1 holds. We let $\delta \rightarrow 1$ such that collusion is not sustainable only if $\bar{m} \geq \bar{x}(k_1, k_n)$.¹⁶ Finally, the analysis above implies that each merger is privately optimal whenever $CS(\hat{p}^*)$ is

¹⁵For example, $(3/6, 3/6)$ and $(2/6, 4/6)$ could result from a remedy of the merger that creates $(1/6, 5/6)$, in which capacity of the merged entity is divested to firm 1 to remedy concerns of unilateral effects. This is similar to what happened in the *Nestlé/Perrier* merger analysed by Compte *et al.* (2002), for example.

¹⁶For $\delta < 1$, the only difference is that there is a discontinuity in $CS(\hat{p}^*)$ at the threshold where collusion is not sustainable. Thus, the line jumps to the $CS(\hat{p}^*)$ associated with the static Nash equilibrium for a lower level of Δm .

strictly lower post-merger than pre-merger.

Each of the plotted lines in Figure 3 has a similar shape. When $CS(\hat{p}^*) = 0$, monitoring is perfect and the average price is p^m . When $CS(\hat{p}^*)$ is upward-sloping, there is imperfect monitoring and the best average price is strictly decreasing in Δm . When $CS(\hat{p}^*)$ is positive and constant, collusion is not sustainable. Furthermore, note that comparing $(1/6, 2/6, 3/6)$ to $(3/6, 3/6)$ in Figure 3 is consistent with moving horizontally from left to right on Figure 2(a) for $\delta \rightarrow 1$, because only the capacity of the smallest firm changes. Likewise, comparing $(1/6, 2/6, 3/6)$ to $(1/6, 5/6)$ is consistent with moving horizontally from left to right on Figure 2(b) for $\delta \rightarrow 1$, because only the capacity of the largest firm changes. For $(2/6, 4/6)$, both the capacities of the smallest and the largest firms are larger than compared with $(1/6, 2/6, 3/6)$.

Figure 3 shows that each merger outcome lowers $CS(\hat{p}^*)$ compared to pre-merger for some levels of Δm . Of particular interest is that the merger that creates a very asymmetric duopoly $(1/6, 5/6)$ can reduce per-unit consumer surplus post-merger, even if it destabilises pre-merger collusion. This occurs at approximately $\Delta m = 0.025$ and the expected per-unit consumer surplus for $(1/6, 5/6)$ can be less than $1/4$ of its value pre-merger. Furthermore, Figure 3 also shows that the merger that creates a symmetric duopoly $(3/6, 3/6)$ can reduce consumer surplus less than the other mergers that create asymmetric duopolies, even though collusion is not sustainable for the latter but it is for the former. For example, at around $\Delta m = 0.09$, the expected per-unit consumer surplus for $(3/6, 3/6)$ is approximately four times its value for $(1/6, 5/6)$ and two times greater than for $(2/6, 4/6)$. This implies, for the merger that creates the very asymmetric duopoly $(1/6, 5/6)$, that it would be appropriate over this range to divest capacity from the merging parties to their smaller rival to remedy concerns of unilateral effects, even if this facilitates collusion. Such remedies would not be implemented if the conventional wisdom were followed.¹⁷ More generally, Figure 3 shows that the competitive prices of asymmetric capacity distributions can be substantially lower than collusive prices of less asymmetric capacity distributions.

5 Extension

Up to this point, consistent with Tirole (1988), we have restricted attention to a setting in which each firm will meet all demand up to its capacity in any given period (see also Campbell *et al.*,

¹⁷Equally, the reverse is also true: for the merger that creates a symmetric duopoly $(3/6, 3/6)$, it would be inappropriate over this range to divest capacity from the merging parties to what would become the largest firm to remedy concerns of coordinated effects.

2005, and Amelio and Biancini, 2010). This is likely to be an appropriate assumption for many markets, including those where collusive prices are agreed at the senior management level, and total output is determined at a lower level by sales representatives (who may be unaware of the collusion).¹⁸ Nevertheless, this is an important assumption because it ensures that a firm cannot hide a deviation from its rivals by limiting how much it sells in an attempt to reduce the resultant expected punishment. This could be achieved if firms were either able to limit the number of units available at the deviation price or able to undercut rivals on only a subset of buyers. This is the issue we explore in this section. In particular, we demonstrate that our main results are robust to this setting under certain conditions.

To adapt our model to such a setting, initially suppose that each firm still sets one price. Buyers can now place orders with any firm with spare capacity and each firm can select how many orders to supply. Demand is again allocated by the proportional rationing rule. Thus, under Assumption 1, it follows that the amount of orders firm i will receive in period t , $d_{it}(p_{it}, \mathbf{p}_{-it}; m_t)$, for any $p_{it} \leq 1$, is:

$$d_{it}(p_{it}, \mathbf{p}_{-it}; m_t) = \begin{cases} k_i & \text{if } p_{it} < p_t^{\max} \\ \min \left\{ \frac{k_i}{K - \sum_{j \in \Omega(p_{it})} k_j} \left(m_t - \sum_{j \in \Omega(p_{it})} s_{jt} \right), k_i \right\} \geq 0 & \text{if } p_{it} = p_t^{\max} \end{cases} \quad (10)$$

In this setting, the residual market demand that is shared between the highest-priced firms is increased if the lowest-priced firms do not supply all orders, $\sum_{j \in \Omega(p_{it})} s_{jt} < \sum_{j \in \Omega(p_{it})} k_j$. Furthermore, suppose that firms may inadvertently overproduce when they try to restrict sales below orders and that it is common knowledge that they can only be sure of limiting their sales to some \underline{k}_i , such that $0 \leq \underline{k}_i < k_i$. Such overproduction may result from over-zealous sales representatives, for example.¹⁹ For simplicity, assume $\underline{k}_i = \gamma k_i$ for all i , where $0 \leq \gamma < 1$, so this extension converges to the main analysis as $\gamma \rightarrow 1$. This then resembles a setting where large firms find it more difficult to limit their sales below orders in absolute terms compared to smaller firms. This may be the case, for example, if each firm must restrict the total output of their sales representatives, who may be separated geographically throughout the market, and larger firms have more sales representatives, which makes this task more problematic. All other assumptions are unchanged.

To establish that there is a public history in this case, consider the history $h^t = (y_0, y_1, \dots, y_{t-1})$

¹⁸This is the setting for many, if not most, cartels (see Harrington, 2006, for evidence from Europe).

¹⁹Indeed, sales representatives were to blame for overproduction in a number of European cartels between 1999-2004 (see Harrington, 2006, p.49-51).

such that, for all $\tau = \{0, 1, \dots, t-1\}$:

$$y_\tau = \begin{cases} \bar{y} & \text{if } d_{i\tau}(p_{i\tau}, \mathbf{p}_{-i\tau}; m_\tau) > d_i^* \forall i \\ \underline{y} & \text{otherwise,} \end{cases}$$

where firm i 's trigger level is $d_i^* = \frac{k_i}{K} m^*(k_l, \bar{m}, \gamma)$ for all i and $m^*(k_l, \bar{m}, \gamma) \equiv \frac{K(\bar{m} - \gamma k_l)}{K-1}$. Such trigger levels are determined by the largest possible orders firms $i \neq l$ receive if all such firms set the same price and firm l undercuts and supplies just γk_l . Such deviations by the smallest firm are most difficult to detect if $\gamma > \frac{\bar{m}}{K}$, so $k_l = k_1$, otherwise it is most difficult to monitor the largest firm, in which case $k_l = k_n$. It then follows from (10) that such trigger levels guarantee that at least one firm will always receive orders below their trigger level, if all firms do not set a common price. This implies that h^t is a public history because each firm knows that all firms' orders will exceed such trigger levels, only if $p_j = p \leq 1$ for all j and if $m > m^*(k_l, \bar{m}, \gamma)$, so $y = \bar{y}$; otherwise, at least one firm's orders will not exceed its trigger level, so $y = \underline{y}$.²⁰

Moving on to the equilibrium payoffs, first notice that the above implies that monitoring is perfect if $m^*(k_l, \bar{m}, \gamma) < \underline{m}$. Otherwise, there is imperfect monitoring, where the probability of observing \underline{y} is:

$$\Pr(\underline{y}|p_i, \mathbf{p}_{-i}) = \begin{cases} G(m^*(k_l, \bar{m}, \gamma)) = \int_{\underline{m}}^{\min\{m^*(k_l, \bar{m}, \gamma), \bar{m}\}} g(m) dm \in [0, 1] & \text{if } p_j = p \forall j \\ 1 & \text{otherwise.} \end{cases} \quad (11)$$

Thus, it follows that $\Pr(\underline{y}|p) < 1$ if and only if $\gamma > \frac{\bar{m}}{K} > 0$. Otherwise, collusion is not sustainable from the fact that $\theta = \beta$, so (6) cannot hold. An implication of this is that deviations by the smallest firm are always most difficult to detect in any SPPE. We can again solve for the set of SPPE payoffs by finding the highest and lowest SPPE values using the constrained optimisation problem described in section 3.2. As before, firm i 's optimal deviation profits from any common price $p \leq 1$ are given by (9), where it will supply its full capacity if it undercuts a sufficiently high collusive price. The reason is that a deviant cannot reduce the resultant expected punishment by limiting its sales below orders. This is due to the fact that the new trigger levels imply that

²⁰Let us briefly return to the case where firms are able to undercut rivals on only a subset of buyers. In such a case, a deviant j could undercut to supply γk_j and also supply its proportion of the residual demand at the collusive price. Nevertheless, this would reduce its rivals' sales further below their trigger levels, d_i^* , than just supplying γk_j . Thus, our analysis here also encompasses the case where firms can charge different prices to different buyers. The reason is that, when firms can charge more than one price, the most devious deviation is to undercut the collusive price to supply γk_j and to charge a price above 1 to all others so they order from other firms.

any such deviations are also now consistent with a bad signal \underline{y} , so they will be punished in the same way as larger deviations.²¹ It also follows from this that we again only need consider the collusive period ICC for some firm i and the punishment period ICC for firm n .

Proposition 6 solves for the highest and lowest SPPE payoffs given there is imperfect monitoring. As before, the set of symmetric SPNE can easily be generated under perfect monitoring by setting $G(m^*(k_1, \bar{m}, \gamma)) = 0$ in the below.

Proposition 6. *For any given $n \geq 2$, $K_{-1} \leq \underline{m} < K$ and $\frac{\bar{m}}{K} < \gamma < 1$, there exists a unique $\underline{x}(k_1, \gamma) \in (\underline{m}, \gamma K)$, that solves $G(m^*(k_1, \underline{x}(k_1, \gamma))) = 0$, and a unique $\bar{x}(k_1, k_n, \gamma) \in (\underline{x}(k_1, \gamma), \gamma K)$, that solves $G(m^*(k_1, \bar{x}(k_1, k_n, \gamma))) = 1 - \frac{k_n}{K} < 1$, such that the highest and lowest SPPE payoffs for firm i are:*

$$k_i \bar{V} = \frac{k_i}{K} \left(\frac{\hat{m} - G(m^*(k_1, \bar{m}, \gamma)) K}{1 - G(m^*(k_1, \bar{m}, \gamma))} \right) \in \left(\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \hat{m} \right) \quad \forall i,$$

$$k_i \underline{V} = k_i \bar{V} - (1 - \delta) \frac{k_i}{k_n} (K - \hat{m}) \in \left(\pi_i^N(k_i, k_n, \hat{m}), k_i \bar{V} \right) \quad \forall i,$$

if and only if $\delta \geq \delta^*(k_1, k_n, \gamma) \equiv \frac{1}{1 - G(m^*(k_1, \bar{m}, \gamma))} \frac{k_n}{K} \in \left(\frac{k_n}{K}, 1 \right)$, for any $\underline{x}(k_1, \gamma) \leq \bar{m} < \bar{x}(k_1, k_n, \gamma)$. Otherwise, collusion is not sustainable.

Punishment periods on the equilibrium path occur more often than in the main analysis, because the trigger levels are higher to ensure firms cannot gain by limiting their sales below orders. It follows from this that the highest SPPE payoffs are lower and the critical discount factor is higher than in the main analysis under imperfect monitoring. Yet, the comparative statics are the same as in section 3.3.²² Finally, the fact that collusion under imperfect monitoring is less profitable in this setting implies that the competitive prices of asymmetric distributions exceeds the collusive prices of a less asymmetric distribution for smaller fluctuations in market demand than in Proposition 5. As a result, the conventional wisdom that coordinated effects are more harmful than unilateral effects is less likely to be true.

²¹However, notice that if the triggers levels were the same as in the main text, then any firm could deviate by limiting their sales below orders.

²²It should be noted that the fact that the smallest firm matters relies on the assumption that small firms can lower their sales more than larger firms. Instead, if $k_i = \underline{k}$ for all i , where $0 \leq \underline{k} \leq k_1$, then the equilibrium profits and the critical discount factor would only depend upon the largest firm. Though following the same steps as set out here, it is easy to see that collusion is sustainable under similar conditions.

6 Concluding Remarks

We have explored the effects of asymmetries in capacity constraints on collusion in a setting where there is demand uncertainty and where firms never directly observe their rivals' prices and sales. Despite the fact that each firm must monitor the collusive agreement using their privately observed prices and sales, we have showed that firms can perfectly detect deviations if demand fluctuations are sufficiently small, and that the critical level is determined by the capacity of the smallest firm. Otherwise, monitoring is imperfect and punishment phases must occur on the equilibrium path. We found that asymmetries between the largest and the smallest firm always hinder collusion. Yet, we also analysed both the unilateral and coordinated effects of mergers in a unified framework. We showed, in contrast to the conventional wisdom, that the competitive prices of asymmetric capacity distributions are substantially higher than the collusive prices of less asymmetric capacity distributions, if demand fluctuations are sufficiently large.

Our results have three implications for merger policy. First, although market transparency is rightly an important criterion in the assessment of coordinated effects in practice, our model re-emphasises the fact that a lack of transparency about rivals' prices and sales is not a sufficient condition to rule out such effects. It is also necessary to check that firms are unable to detect deviations using only their own sales. Second, while the possible effects of imperfect monitoring are explicitly mentioned in general terms in the most recent US and European horizontal merger guidelines, our model suggests that such monitoring will be more difficult if the market structure is more asymmetric. Finally, symmetric merger outcomes where collusion is thought to be a problem should not be presumed to be more harmful than asymmetric merger outcomes where collusion is not considered a problem. A collusive agreement may require sufficiently frequent price wars that actually lead to higher consumer surplus than compared to an alternative outcome in which one firm's market power is strengthened unilaterally. This is more likely, according to our model, when market demand fluctuates to a large extent over time.

References

- [1] Abreu, D., Pearce, D. and Stacchetti, E. (1986) "Optimal Cartel Equilibria with Imperfect Monitoring," *Journal of Economic Theory*, 39(1), 251-269
- [2] Abreu, D., Pearce, D. and Stacchetti, E. (1990) "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica*, 58(5), 1041-1063

- [3] Amelio, A., and Biancini, S. (2010) “Alternating Monopoly and Tacit Collusion,” *Journal of Industrial Economics*, 58(2), 402-423
- [4] Bos, I. and Harrington, J. (2010) “Endogenous Cartel Formation with Heterogeneous Firms,” *RAND Journal of Economics*, 41(1), 92-117
- [5] Bos, I. and Harrington, J. (2014) “Competition Policy and Cartel Size,” *International Economic Review*, forthcoming
- [6] Campbell, C., Ray, G., and Muhanna, W. (2005) “Search and Collusion in Electronic Markets,” *Management Science*, 51, 497-507
- [7] Compte, O., Jenny, F. and Rey, P. (2002) “Capacity Constraints, Mergers and Collusion,” *European Economic Review*, 46(1), 1-29
- [8] Dasgupta, P. and Maskin, E. (1986) “The Existence of Equilibrium in Discontinuous Economic Games, II: Applications” *Review of Economic Studies*, 53(1), 27-41
- [9] Davies, S. and Olczak, M. (2010) “Assessing the Efficacy of Structural Merger Remedies: Choosing Between Theories of Harm?” *Review of Industrial Organization*, 37(2), 83-99
- [10] Davies, S., Olczak, M., and Coles, H. (2011) “Tacit Collusion, Firm Asymmetries and Numbers: Evidence from EC Merger Cases,” *International Journal of Industrial Organization*, 29(2), 221-31
- [11] Fabra, N. (2006) “Collusion with Capacity Constraints over the Business Cycle,” *International Journal of industrial Organization*, 24(1), 69-81
- [12] Farrell, J. and Shapiro, C. (1990) “Horizontal Mergers: An Equilibrium Analysis,” *American Economic Review*, 80(1), 107-126
- [13] Fonseca, M. and Normann, H.-T. (2008) “Mergers, Asymmetries and Collusion: Experimental Evidence,” *Economic Journal*, 118(1), 287-400
- [14] Fonseca, M. and Normann, H.-T. (2012) “Explicit vs. Tacit Collusion – The Impact of Communication in Oligopoly Experiments,” *European Economic Review*, 56(8), 1759-1772
- [15] Fudenberg, D. and Tirole, J. (1994), *Game Theory*, MIT Press
- [16] Gal-Or, E. (1984) “Price Dispersion with Uncertain Demand,” *International Economic Review*, 25(2), 441-457

- [17] Green, E. and Porter, R. (1984) “Non-Cooperative Collusion under Imperfect Price Information,” *Econometrica*, 52, 87-100
- [18] Harrington, J. (2006) “How do Cartels Operate?” *Foundations and Trends in Microeconomics*, 2(1), 1-105
- [19] Harrington, J. and Skrzypacz, A. (2007) “Collusion under Monitoring of Sales,” *RAND Journal of Economics*, 38(2), 314-331
- [20] Harrington, J. and Skrzypacz, A. (2011) “Private Monitoring and Communication in Cartels: Explaining Recent Collusive Practices,” *American Economic Review*, 101(6), 2425-2449
- [21] Ivaldi, M., Jullien, B., Rey, P., Seabright, P., and Tirole, J. (2003a) “The Economics of Tacit Collusion,” Report for DG Comp, European Commission
- [22] Ivaldi, M., Jullien, B., Rey, P., Seabright, P., and Tirole, J. (2003b) “The Economics of Unilateral Effects,” Report for DG Comp, European Commission
- [23] Knittel, C. and Lepore, J. (2010) “Tacit Collusion in the Presence of Cyclical Demand and Endogenous Capacity Levels,” *International Journal of Industrial Organization*, 28(2), 131-144
- [24] Kühn, K.-U., (2001) “An Economist’s Guide through the Joint Dominance Jungle,” John M. Olin Centre for Law and Economics Working Paper 02-014
- [25] Kühn, K.-U., (2004) “The Coordinated Effects of Mergers in Differentiated Product Markets,” John M. Olin Centre for Law and Economics Working Paper 04-020
- [26] Lambson, E. (1994) “Some Results on Optimal Penal Codes in Asymmetric Bertrand Supergames,” *Journal of Economic Theory*, 62(2), 444-468
- [27] Lyons, B. (2002) “Could Politicians be More Right than Economists? A Theory of Merger Standards,” Centre for Competition and Regulation Working Paper 02-1
- [28] Motta, M., Polo, M., and Vasconcelos, H. (2003). “Merger Remedies in the European Union: An Overview,” in Lévêque, F. and Shelanski, H., (eds.), *Merger Remedies in American and European Union Competition Law*, Edward Elgar
- [29] Röller, L.-H. and Mano, M. (2006) “The Impact of the New Substantive Test in European Merger Control,” *European Competition Journal*, 2(1), 9-28

- [30] Staiger, R. and Wolak, F. (1992) “Collusive Pricing with Capacity Constraints in the Presence of Demand Uncertainty,” *RAND Journal of Economics*, 23(2), 203-220
- [31] Stigler, G. (1964) “A Theory of Oligopoly,” *Journal of Political Economy*, 72, 44–61
- [32] Tirole, J. (1988) *The Theory of Industrial Organisation*, MIT Press
- [33] Vasconcelos, H. (2005) “Tacit Collusion, Cost Asymmetries, and Mergers,” *RAND Journal of Economics*, 36(1), 39-62

Appendix A

Proof of Lemma 1. There exists a unique pure strategy Nash equilibrium if $\underline{m} \geq K$, where $\pi_i^N = k_i \forall i$. This follows from $\pi_i(p_i, \mathbf{p}_{-i}) = p_i k_i \forall p_i \leq 1$, so the best reply of firm i is $p_i = 1$ for any $\mathbf{p}_{-i}, \forall i$. There is no pure strategy Nash equilibrium if $\underline{m} < K$. To see this, note that any such candidate equilibrium requires $p_j = p \forall j$. Otherwise, firm $i \in \Omega(p^{\max})$ has an incentive to increase its price towards p^{\max} , from $\pi_i(p_i, \mathbf{p}_{-i}) = p_i k_i \forall p_i < p^{\max}$. However, for any $p \in (0, 1]$, firm i has an incentive to lower its price, since $\pi_i(p - \epsilon, p) > \pi_i(p)$ if $\underline{m} < K$, where $\epsilon > 0$ but small. Moreover, for $p = 0$, firm i has an incentive to raise its price, since Assumption 1 ensures $\pi_i(\epsilon, 0) > 0 \forall i$.

Nevertheless, if $K > \underline{m} \geq K_{-1}$, the existence of a mixed strategy Nash equilibrium is guaranteed by Theorem 1 of Dasgupta and Maksin (1986). To characterise this equilibrium, let $H_i(p)$ denote the probability that firm i charges a price less than or equal to p . Below we demonstrate that the equilibrium profits are given by (2) for all i and that:

$$H_i(p) = \frac{1}{k_i} \left[\frac{(\bar{\pi}_n - p k_n)}{p k_n \left(\int_{\underline{m}}^{\min\{K, \bar{m}\}} (m - K) g(m) dm \right) \prod_{j=1}^n k_j} \right]^{1/(n-1)}, \quad (12)$$

where firm i 's expected profits are given by $\bar{\pi}_i$ in (3), if it is strictly the highest-priced firm with $p_i = 1$. This converges to the analysis in Fonseca and Normann (2008) as $\bar{m} \rightarrow \underline{m}$.

In equilibrium, firm i must receive the following expected profit from charging $p \leq 1$:

$$p \left(\prod_{j \neq i} H_j(p) \bar{\pi}_i + \left(1 - \prod_{j \neq i} H_j(p) \right) k_i \right) = \frac{k_i}{k_n} \bar{\pi}_n, \quad \forall i \quad (13)$$

where $\prod_{j \neq i} H_j(p)$ is the probability that firm i is the highest-priced firm. To solve for the right-hand side of (13), notice firm i has no incentive to price below $\bar{\pi}_i/k_i \equiv \underline{p}_i$, where $\underline{p}_n \geq \underline{p}_{n-1} \geq$

$\dots \geq \underline{p}_1$. Moreover, any firm $j < n$ can guarantee profits of $\frac{k_j}{k_n} \bar{\pi}_n \geq \bar{\pi}_j$ by charging a price marginally below \underline{p}_n , so all firms have no incentive to price below \underline{p}_n . Finally, the fact that all firms $j < n$ place positive probability on charging \underline{p}_n is necessary and sufficient to ensure \underline{p}_n is also the lowest price that firm n will charge. Thus, the lower bound of $H_i(p)$ is $\underline{p} = \underline{p}_n = \bar{\pi}_n/k_n \forall i$. Manipulating (13) yields:

$$H_i(p) = \frac{pk_n(\bar{\pi}_i - k_i)}{\bar{\pi}_n - pk_n} \prod_j H_j(p) \frac{1}{k_i}. \quad (14)$$

Noting that $\bar{\pi}_i - k_i = \int_{\underline{m}}^{\min\{K, \bar{m}\}} (m - K) g(m) dm \forall i$ for any $K > \underline{m}$ from (3), it follows from (14) that:

$$\prod_j H_j(p) = \left[\frac{pk_n \left(\int_{\underline{m}}^{\min\{K, \bar{m}\}} (m - K) g(m) dm \right)}{\bar{\pi}_n - pk_n} \prod_j H_j(p) \right]^n \prod_{l=1}^n \left(\frac{1}{k_l} \right).$$

Thus, solving for $\prod_j H_j(p)$ and substituting into (14) shows that $H_i(p)$ is as claimed in (12).

It follows from (12) that $H_i(1) \leq 1$ if $\frac{k_i^{n-1}}{\prod_{j \neq n} k_j} \geq 1$. This has two implications. First, if $\frac{k_i^{n-1}}{\prod_{j \neq n} k_j} \geq 1$, then firm i randomises over $[\underline{p}, 1]$ and puts mass of $1 - H_i(1)$ on a price of 1 when the inequality is strict. Note that $\frac{k_i^{n-1}}{\prod_{j \neq n} k_j} > 1$ never holds if $k_i = k \forall i$ but always holds for firm n if $k_n > k_1$. Second, if $\frac{k_i^{n-1}}{\prod_{j \neq n} k_j} < 1$ for some $i < n$, then firm i randomises over $[\underline{p}, \bar{p}_i]$ where $\bar{p}_i < 1$ solves $H_i(\bar{p}_i) = 1$. Consequently, the probability distributions of the larger firms with higher upper bounds must be adjusted accordingly. For example, if $\bar{p}_i < 1$ only for firm 1 (which is the case for any triopoly with $k_1 < k_2$), then the largest $n - 1$ firms play with the $H_i(p)$ adjusted so that $n - 1$ replaces n over $[\bar{p}_1, 1]$. Note that $\frac{k_i^{n-1}}{\prod_{j \neq n} k_j} < 1$ never holds if $n = 2$ or if $k_i = k \forall i$ for any $n \geq 2$. ■

Proof of Proposition 1. There is perfect monitoring if $\underline{m} > m^*(k_1, \bar{m})$ and imperfect monitoring otherwise. Given $\frac{\partial m^*}{\partial \bar{m}} > 0$, it follows that there is a unique level of \bar{m} that solves $m^*(k_1, \bar{m}) = \underline{m}$. Substituting in for $m^*(k_1, \bar{m})$ and rearranging yields $\bar{m} = k_1 + \underline{m} \frac{K-1}{K} \equiv \underline{x}(k_1)$, where $\underline{x}(k_1) \in (\underline{m}, K)$ for any $\underline{m} < K$. Thus, monitoring is perfect if $\bar{m} < \underline{x}(k_1)$, as this implies $\underline{m} > m^*(k_1, \bar{m})$. Otherwise, there is imperfect monitoring. ■

Proof of Proposition 2. The Lagrangean function for our constrained maximisation problem is:

$$L = \bar{V} + \lambda \xi_i^c + \mu \xi_n^p$$

where ξ_i^c and ξ_n^p denote the slack in the collusive phase and the punishment phase ICCs for firm n , respectively, such that

$$\xi_i^c \equiv (1 - \delta) \left(-(\pi_i(p_i^*, p^c) - \pi_i(p^c)) + \delta(\beta - \alpha) (1 - \Pr(\underline{y}|p)) [\pi_i(p^c) - \pi_i(p^p)] \right)$$

$$\xi_n^p \equiv (1 - \delta) \left(-(\pi_n(p_i^*, p^p) - \pi_n(p^p)) + \delta(\beta - \alpha) (1 - \Pr(\underline{y}|p)) [\pi_n(p^c) - \pi_n(p^p)] \right).$$

and where

$$\bar{V} = (1 - \delta) \frac{\hat{m}}{K} p^c + \delta \left[\theta \frac{\hat{m}}{K} p^p + (1 - \theta) \frac{\hat{m}}{K} p^c \right]$$

$$\underline{V} = (1 - \delta) \frac{\hat{m}}{K} p^p + \delta \left[\theta \frac{\hat{m}}{K} p^p + (1 - \theta) \frac{\hat{m}}{K} p^c \right].$$

We proceed by solving the constrained maximum for a given p^c , to see explicitly how the expected profits and critical discount factor vary with p^c . Thus, the Kuhn-Tucker conditions for a maximum are:

$$\begin{aligned} \frac{\partial L}{\partial z} = \frac{\partial \bar{V}}{\partial z} + \lambda \frac{\partial \xi^c}{\partial z} + \mu \frac{\partial \xi_n^p}{\partial z} &\leq 0, \quad z \geq 0, \quad z \frac{\partial L}{\partial z} = 0 \quad \text{for } z = \alpha, \beta \\ \frac{\partial L}{\partial p^p} = \frac{\partial \bar{V}}{\partial p^p} + \lambda \frac{\partial \xi^c}{\partial p^p} + \mu \frac{\partial \xi_n^p}{\partial p^p} &= 0 \\ \frac{\partial L}{\partial \lambda} = \xi^c \geq 0, \quad \lambda \geq 0, \quad \lambda \frac{\partial L}{\partial \lambda} &= 0 \\ \frac{\partial L}{\partial \mu} = \xi_n^p \geq 0, \quad \mu \geq 0, \quad \mu \frac{\partial L}{\partial \mu} &= 0. \end{aligned}$$

We begin by establishing that the Kuhn-Tucker conditions are satisfied if $\frac{\partial L}{\partial \lambda} = \xi^c = 0$ and $\frac{\partial L}{\partial \mu} = \xi_n^p \geq 0$ where $\alpha^* = 0$ and $\beta^* > 0$. First, notice $\frac{\partial L}{\partial \alpha} = \frac{\partial \bar{V}}{\partial \alpha} + \lambda \frac{\partial \xi^c}{\partial \alpha} + \mu \frac{\partial \xi_n^p}{\partial \alpha} < 0$ from $\frac{\partial \bar{V}}{\partial \alpha} < 0$, $\frac{\partial \xi^c}{\partial \alpha} < 0$, $\frac{\partial \xi_n^p}{\partial \alpha} < 0$, $\lambda \geq 0$ and $\mu \geq 0$. So, $\alpha^* = 0$. Furthermore, $\beta > 0$ is a necessary condition for $\frac{\partial L}{\partial \lambda} = \xi^c \geq 0$ and $\frac{\partial L}{\partial \mu} = \xi_n^p \geq 0$, and this implies $\frac{\partial L}{\partial \beta} = 0$. Solving for the latter and rearranging shows $\lambda = \frac{1}{k_i(1-\delta)} \frac{\Pr(\underline{y}|p)}{1-\Pr(\underline{y}|p)} - \frac{k_n}{k_i} \mu$. Substituting this into $\frac{\partial L}{\partial p^p} = 0$ yields $\mu = 0$, so $\lambda = \frac{1}{k_i(1-\delta)} \frac{\Pr(\underline{y}|p)}{1-\Pr(\underline{y}|p)} > 0$. These values imply $\frac{\partial L}{\partial \mu} = \xi_n^p \geq 0$ and $\frac{\partial L}{\partial \lambda} = \xi^c = 0$, respectively, where $\xi^c = 0$ if $\beta = \frac{1}{\delta(1-\Pr(\underline{y}|p))} \left(\frac{\pi_i(p_i^*, p^c) - \pi_i(p^c)}{\pi_i(p^c) - \pi_i(p^p)} \right) \equiv \beta^* > 0$.

To solve for \bar{V} and \underline{V} , first substitute β^* and α^* into \bar{V} to get $\bar{V} = \frac{1}{K} \left(\frac{\hat{m} - G(m^*)K}{1 - G(m^*)} \right) p^c$, where $\Pr(\underline{y}|p) = G(m^*)$ from (5). This is maximised for $p^c = 1$, so $k_i \bar{V}$ is as claimed. Substituting this into \underline{V} then yields $k_i \underline{V} = k_i \bar{V} - (1 - \delta) \frac{k_i}{k_n} (K - \hat{m})$. Furthermore, $\delta^*(k_1, k_n)$ is obtained from the fact that $\beta^* \leq 1$ if $\delta \geq \frac{1}{(1 - G(m^*))} \frac{K - \hat{m}}{\hat{m}(p^c - p^p)}$. This is minimised when p^p is set as low as possible to the extent that $\xi_n^p = 0$. Notice $\xi_i^p > \xi_i^c$ if $p^p > \frac{\bar{\pi}_i}{k_i}$, which implies $\xi_n^p > \xi_n^c = 0 \forall p^p > \frac{\bar{\pi}_n}{k_n} = \underline{p}$. Thus, consider $p^p \leq \underline{p}$ where $\pi_n(p_i^*, p^p) = \bar{\pi}_n$ from (9). Substituting β^* and α^* into ξ_n^p shows $\xi_n^p \geq 0$ if $p^p \geq p^c - \frac{K}{\hat{m}} (p^c - \underline{p}) \equiv p^*$, where $p^* < \underline{p} \forall p^c > \underline{p}$ and where $p^* \geq 0$ if

$\hat{m} \geq \frac{K^2}{K+k_n}$. Letting $p^p = p^*$ such that $\xi_n^p = 0$ yields $\beta^* \leq 1$ if $\delta \geq \frac{1}{(1-G(m^*))} \frac{K-\hat{m}}{K(p^c-p)}$. The latter is minimised for $p^c = 1$, so $\delta^*(k_1, k_n)$ is as claimed, where $\delta^*(k_1, k_n) < 1$ if $G(m^*) < 1 - \frac{k_n}{K}$.

Finally, it follows from $\frac{\partial G(m^*)}{\partial \bar{m}} > 0$ that there is a unique level of \bar{m} , denoted $\bar{x}(k_1, k_n)$, that sets $G(m^*(k_1, \bar{m})) = 1 - \frac{k_n}{K} < 1$, where $\bar{x}(k_1, k_n) < K$ and where $G(m^*(k_1, \bar{m})) \in [0, 1 - \frac{k_n}{K})$ for all $\bar{m} \in [\underline{x}(k_1), \bar{x}(k_1, k_n))$. This implies $\delta^*(k_1, k_n) \in (\frac{k_n}{K}, 1)$, $k_i \bar{V} \in (\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \hat{m})$ and $k_i \underline{V} \in (\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \bar{V})$ for all $\bar{m} \in [\underline{x}(k_1), \bar{x}(k_1, k_n))$. ■

Proof of Proposition 3. Differentiating $\delta^*(k_1, k_n) = \frac{1}{(1-G(m^*(k_1, \bar{m})))} \frac{k_n}{K}$ with respect to k_j yields:

$$\frac{\partial \delta^*}{\partial k_j} = \frac{1}{K[1-G(m^*)]} \left[\frac{\partial k_n}{\partial k_j} + k_n \frac{g(m^*)}{1-G(m^*)} \frac{\partial m^*}{\partial k_j} \right].$$

Thus, $\frac{\partial \delta^*}{\partial k_1} < 0$ from $\frac{\partial k_n}{\partial k_1} \in [-1, 0]$ and $\frac{\partial m^*}{\partial k_1} = -\frac{K(K-\bar{m})}{(K-k_1)^2} < 0$, and $\frac{\partial \delta^*}{\partial k_n} > 0$ from $\frac{\partial k_n}{\partial k_n} = 1$ and $\frac{\partial m^*}{\partial k_n} = 0$. Finally, $\delta^*(k_n) = \frac{k_n}{K}$ implies $\frac{\partial \delta^*}{\partial k_n} > 0$. ■

Proof of Proposition 4. It follows that:

$$\hat{p}^c(k_1, \bar{m}) = \frac{K}{\hat{m}} \bar{V} = \frac{\hat{m} - G(m^*(k_1, \bar{m}))K}{\hat{m}(1-G(m^*(k_1, \bar{m})))},$$

where $\hat{p}^c(k_1, \bar{m}) \in (\hat{p}^N(k_n, \hat{m}), p^m)$ from $k_i \bar{V} \in (\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \hat{m})$. Differentiating $\hat{p}^c(k_1, \bar{m})$ with respect to k_j yields:

$$\frac{\partial \hat{p}^c}{\partial k_j} = -\frac{(K-\hat{m})g(m^*)}{\hat{m}(1-G(m^*))^2} \frac{\partial m^*}{\partial k_1} \frac{\partial k_1}{\partial k_j}.$$

Thus, $\frac{\partial \hat{p}^c}{\partial k_1} > 0$ since $0 < \hat{m} < \bar{m} < K$, $\frac{\partial m^*}{\partial k_1} < 0$ and $\frac{\partial k_1}{\partial k_1} = 1$, and $\frac{\partial \hat{p}^c}{\partial k_j} \leq 0$ for $j \neq 1$ from $\frac{\partial k_1}{\partial k_j} \in [-1, 0]$, where $\frac{\partial \hat{p}^c}{\partial k_j} = 0$ if $\frac{\partial k_1}{\partial k_j} = 0$. ■

Proof of Proposition 5. We first show that $\hat{p}^N(k'_n, \hat{m}) > \hat{p}^c(k_1, \bar{m})$ if $\bar{m} > \bar{x}(k_1, k'_n)$. This follows since $\hat{p}^N(k'_n, \hat{m}) > \hat{p}^c(k_1, \bar{m})$ if $G(m^*(k_1, \bar{m})) > 1 - \frac{k'_n}{K}$. In Proposition 2, $\bar{x}(k_1, k_n)$ is defined as the level of \bar{m} that solves $G(m^*(k_1, \bar{x}(k_1, k_n))) = 1 - \frac{k_n}{K}$. Thus, $G(m^*(k_1, \bar{x}(k_1, k'_n))) = 1 - \frac{k'_n}{K}$. This and $\frac{\partial G(m^*)}{\partial \bar{m}} > 0$ implies $G(m^*(k_1, \bar{m})) > 1 - \frac{k'_n}{K}$ for any $\bar{m} > \bar{x}(k_1, k'_n)$, where $\hat{p}^N(k'_n, \hat{m}) > \hat{p}^c(k_1, \bar{m})$. This comparison is only meaningful if $\bar{m} < \bar{x}(k_1, k_n)$ and if $\delta \geq \delta^*(k_1, k_n)$ such that $\hat{p}^c(k_1, \bar{m})$ is an equilibrium average price. So, we next show that $\bar{x}(k_1, k'_n) < \bar{x}(k_1, k_n)$ if $k'_n > k_n$. Using the implicit function theorem on $Z \equiv 1 - \frac{k_n}{K} - G(m^*(k_1, \bar{m})) = 0$ yields:

$$\frac{\partial \bar{x}}{\partial k_j} = -\frac{\frac{\partial Z}{\partial k_j}}{\frac{\partial Z}{\partial \bar{m}}} = -\frac{1}{(g(m^*) \frac{\partial m^*}{\partial \bar{m}})} \left(\frac{1}{K} \frac{\partial k_n}{\partial k_j} + g(m^*) \frac{\partial m^*}{\partial k_1} \frac{\partial k_1}{\partial k_j} \right).$$

Thus, $\frac{\partial \bar{x}}{\partial k_n} < 0$ from $\frac{\partial m^*}{\partial \bar{m}} > 0$, $\frac{\partial k_n}{\partial k_n} = 1$, $\frac{\partial m^*}{\partial k_1} < 0$ and $\frac{\partial k_1}{\partial k_n} \in [-1, 0]$. So, $\bar{x}(k_1, k'_n) < \bar{x}(k_1, k_n)$ if $k'_n > k_n$. Thus, the above implies that if $k'_n > k_n$, then $\hat{p}^N(k'_n, \hat{m}) > \hat{p}^c(k_1, \bar{m})$, if $\bar{x}(k_1, k'_n) < \bar{m} < \bar{x}(k_1, k_n)$ for any $\delta \geq \delta^*(k_1, k_n)$. Finally, notice that collusion is not sustainable for (k'_1, k'_n) for any $\bar{x}(k_1, k'_n) < \bar{m} < \bar{x}(k_1, k_n)$ from Proposition 2, because $\bar{x}(k'_1, k'_n) \leq \bar{x}(k_1, k'_n) < \bar{m}$ from $\frac{\partial \bar{x}}{\partial k_1} > 0$. ■

Proof of Proposition 6. The Lagrangean function and the Kuhn-Tucker conditions for this constrained maximisation problem are the same as in the proof of Proposition 2. The only change is that $\Pr(y|p) = G(m^*(k_1, \bar{m}, \gamma))$ from (11). Thus, given the proof of Proposition 2 is initially written in terms of $\Pr(y|p)$, it follows immediately that the Kuhn-Tucker conditions are satisfied if $\frac{\partial L}{\partial \lambda} = \xi_i^c = 0$ and $\frac{\partial L}{\partial \mu} = \xi_n^p \geq 0$ where $\alpha^* = 0$ and $\beta^* = \frac{1}{\delta(1-\Pr(y|p))} \left(\frac{\pi_i(p_i^*, p^c) - \pi_i(p^c)}{\pi_i(p^c) - \pi_i(p^p)} \right) > 0$. Following the other steps of the proof of Proposition 2, but with $\Pr(y|p) = G(m^*(k_1, \bar{m}, \gamma))$ in this case, shows that $k_i \bar{V}$, $k_i \underline{V}$ and $\delta^*(k_1, k_n, \gamma)$ are as claimed.

Furthermore, monitoring is perfect if $m^*(k_1, \bar{m}, \gamma) < \underline{m}$, which implies that there is imperfect monitoring if $\bar{m} \geq \underline{m} \frac{K-1}{K} + \gamma k_1 \equiv \underline{x}(k_1, \gamma)$, where $G(m^*(k_1, \underline{x}(k_1, \gamma), \gamma)) = 0$ and $\underline{m} < \underline{x}(k_1, \gamma) < \gamma K$. Finally, from $\frac{\partial G(m^*)}{\partial \bar{m}} > 0$ there is a unique level of \bar{m} , denoted $\bar{x}(k_1, k_n, \gamma)$, that sets $G(m^*(k_1, \bar{m}, \gamma)) = 1 - \frac{k_n}{K} < 1$, where $\bar{x}(k_1, k_n, \gamma) < \gamma K$ and where $G(m^*(k_1, \bar{m}, \gamma)) \in [0, 1 - \frac{k_n}{K}]$ for all $\bar{m} \in [\underline{x}(k_1, \gamma), \bar{x}(k_1, k_n, \gamma)]$. This implies $\delta^*(k_1, k_n, \gamma) \in (\frac{k_n}{K}, 1)$, $k_i \bar{V} \in (\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \hat{m})$ and $k_i \underline{V} \in (\pi_i^N(k_i, k_n, \hat{m}), \frac{k_i}{K} \bar{V})$ for all $\bar{m} \in [\underline{x}(k_1, \gamma), \bar{x}(k_1, k_n, \gamma)]$. ■

Appendix B

To check the robustness of our main results, we solve the game following the approach of Tirole (1988). In his setting, there are two symmetric firms selling a homogeneous product, without capacity constraints, and there is a chance that market demand is either high or zero. There is imperfect monitoring, because a firm cannot be sure that making zero sales is caused by low market demand or is due to a deviation by its rival. Consequently, firms follow a strategy profile in which they set the monopoly price until at least one firm receives zero sales. Then they play the static Nash equilibrium for T periods, after which they return to setting the monopoly price and the sequence repeats.²³

²³Notice that the event that triggers the T period punishment is common knowledge. If a firm makes zero sales because market demand is zero, its rival will also make zero sales. In contrast, if a firm makes zero sales because its rival deviated and demand is high, then the deviant makes twice the sales it would have had it set the collusive

To replicate this approach, we consider the following strategy profile, which we refer to as the Tirole (1988) strategy profile. There are “collusive phases” and “punishment phases”. In a collusive phase, firms sets the monopoly price, $p^m = 1$. If $y_t = \bar{y}$ such that all firms received sales above their trigger levels, the collusive phase continues into the next period $t + 1$. If $y_t = \underline{y}$ such that at least one firm received sales below its trigger level, firms enter a punishment phase in the next period, in which they play the static Nash equilibrium for T periods, after which a new collusive phase begins and the sequence repeats. In contrast to the analysis in section 3.2, the Tirole (1988) strategy profile is not strongly symmetric, it does not require a public randomisation device, and it always has prices above marginal cost in a punishment phase. Finally, we impose $K_{-1} \leq \underline{m} < K$ such that Assumption 1 is satisfied and the Nash equilibrium is in mixed strategies. We also let $\underline{m} < \underline{x}(k_1) \leq \bar{m}$ to restrict attention to imperfect monitoring, since it is trivial to see that our results hold for trigger strategies under perfect monitoring.

Denoting firm i 's expected (normalised) profit in a collusive phase as v_i^c and its expected (normalised) profit at the start of a punishment phase as v_i^p , it follows that:

$$\begin{aligned} v_i^c &= (1 - \delta) \pi_i(p^m) + \delta [(1 - G(m^*(k_1, \bar{m}))) v_i^c + G(m^*(k_1, \bar{m})) v_i^p] \\ v_i^p &= (1 - \delta) \sum_{t=0}^{T-1} \delta^t \pi_i^N(k_i, k_n, \hat{m}) + \delta^T v_i^c \end{aligned}$$

where $\pi_i(p^m) > v_i^c > v_i^p$ for any $T > 0$ and where $v_i^p > \pi_i^N(k_i, k_n, \hat{m})$ for any $T < \infty$. The Tirole (1988) profile of strategies is a PPE if, for each date t and any history h^t , the strategies yield a Nash equilibrium from that date on. Since firms play the static Nash equilibrium during each period of the punishment phase, it is clear that they have no incentive to deviate in any such periods. Thus, we need only consider deviations during a collusive phase.

Firm i will not deviate in any collusive phase if it cannot gain by marginally undercutting p^m to supply its full capacity k_i . This provides the following ICC for firm i :

$$v_i^c \geq (1 - \delta) \pi_i^d + \delta v_i^p. \quad (15)$$

where $\pi_i^d = k_i > \pi_i(p^m)$ for all $\underline{m} < K$. Notice that (15) is never satisfied for any $\bar{m} \geq K$, as then $G(m^*(k_1, \bar{m})) = 1$ from (5). Thus, we can henceforth focus on the case where $\bar{m} < K$. Substituting v_i^p into v_i^c and solving yields:

$$v_i^c = \pi_i^N(k_i, k_n, \hat{m}) + \frac{(1 - \delta)}{1 - \delta + G(m^*(k_1, \bar{m})) \delta (1 - \delta^T)} (\pi_i(p^m) - \pi_i^N(k_i, k_n, \hat{m})).$$

It is then helpful to let $v_i^c = k_i V^c$, such that $V^c \equiv \frac{1}{K} \sum_i v_i^c$, so we can rewrite (15) as:

$$k_i V^c (1 - \delta^{T+1}) \geq (1 - \delta) k_i + \delta (1 - \delta^T) \frac{k_i}{k_n} (\hat{m} - K_{-n}). \quad (16)$$

price, and from this information it can infer that its rival made no sales.

Since the k_i 's cancel it follows that if the collusive period ICC holds for firm i , then it also holds for all other firms $j \neq i$. Substituting $k_i V^c$ into (16) and rearranging yields:

$$\delta^T \leq \frac{k_n - \delta K (1 - G(m^*(k_1, \bar{m})))}{\delta [k_n - K (1 - G(m^*(k_1, \bar{m})))]}. \quad (17)$$

It follows from this that firms will not deviate from p^m in a collusive phase for a sufficiently large T , if both the numerator and the denominator of (17) are negative. This is true if $\delta \geq \frac{1}{(1 - G(m^*(k_1, \bar{m})))} \frac{k_n}{K} = \delta^*(k_1, k_n)$ and if $G(m^*(k_1, \bar{m})) < 1 - \frac{k_n}{K}$, where $G(m^*(k_1, \bar{m})) < 1 - \frac{k_n}{K}$ ensures $\delta^*(k_1, k_n) < 1$.

Thus, similar to Tirole (1988), there are three necessary conditions that must be satisfied. First, the length of the punishment phase must be sufficiently long, where the critical length, denoted $T^*(k_1, k_n)$, is implicitly defined by the level of T where (17) holds with equality. Second, firms must also be sufficiently patient, where $T^*(k_1, k_n) \rightarrow \infty$ if $\delta = \delta^*(k_1, k_n)$ and $T^*(k_1, k_n) < \infty$ for any $\delta > \delta^*(k_1, k_n)$. This implies that even a punishment phase that lasts an infinite number of periods is insufficient to outweigh the short-term benefit from deviating, if firms are not sufficiently patient. Third, the maximum market demand must be sufficiently low, otherwise $G(m^*(k_1, \bar{m}))$ so high that (17) cannot hold for any δ and T . So, the level of \bar{m} that sets $\delta^*(k_1, k_n)$ equal to 1 implicitly defines the critical threshold, $\bar{x}(k_1, k_n) < K$, where $G(m^*(k_1, \bar{x}(k_1, k_n))) = 1 - \frac{k_n}{K} < 1$. The latter two conditions are the same as in Proposition 2, and the first condition plays a similar role as β^* in the main text.

Finally, given $k_i V^c$ is strictly decreasing in T , the highest equilibrium payoffs can be found by evaluating $k_i V^c$ at $T^*(k_1, k_n)$. Thus, noting from (17) that:

$$1 - \delta^{T^*} = \frac{-(1 - \delta)k_n}{\delta [k_n - K (1 - G(m^*(k_1, \bar{m})))]},$$

we can obtain:

$$k_i V^c = \frac{k_i}{K} \left(\frac{\hat{m} - G(m^*(k_1, \bar{m})) K}{1 - G(m^*(k_1, \bar{m}))} \right) = k_i \bar{V} \quad \forall i.$$

This is the same as the highest SPPE payoffs in Proposition 2. Thus, the comparative statics and the implications for mergers are the same for the Tirole (1988) strategy profile as in section 3.3 and 3.4.