Prizes versus Contracts as Incentives for Innovation*

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Abstract

The procurement of an innovation involves motivating a research effort to generate a new idea and implementing that idea efficiently. If research efforts are unverifiable and implementation costs are private information, a trade-off arises between the two objectives. The optimal mechanism resolves the tradeoff via two instruments: a monetary prize and the contract for implementing the project. When a proposal receives a high evaluation by the buyer, the innovator obtains a monetary reward, and the allocation of contract rights is distorted in his favor. Otherwise, the innovator receives no reward and can even be handicapped in the allocation of contract rights. In particular, if firms' private information on their implemention costs is not project-specific, then the project can be selected prior to (and thus independently from) choosing the implementor; otherwise, the selection of the project and the choice of the implementor should be made jointly.

JEL Classification:

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1 Introduction

When buyers have specific needs that the products and services available on the market cannot readily satisfy, they may procure an innovation. This is however considerably more involved than procuring a readily available product. While the latter can simply focus on identifying the most qualified provider (i.e., highest quality or lowest cost, or some happy compromise), the former involves identifying and motivating a firm to engage in costly research to develop an idea and subsequently to identify a firm (possibly the same or a different one) to implement the idea. A number of questions arise as to how to incentivize a potential innovator: for instance, should the buyer offer a prize? Or should the buyer favor the innovator when allocating the contract for implementing the project?

These questions have triggered intense debates among academics and policy makers alike. In particular, using contract rights as incentives for innovation remains controversial. If an innovator happens to be the most efficient at implementing the innovation, it should clearly be awarded the contract to do so. But there appears to be no clear justification for favoring the innovator at the implementation stage, above and beyond the level warranted by efficient allocation. Indeed, intuition seems to suggest that any incentive the buyer can provide via distorting contract allocation can be provided more efficiently with pure monetary prizes. The main purpose of our study is to investigate these questions.

The lack of consensus on this issue is apparent in the treatment of unsolicited proposals. According to Hodges and Dellacha (2007), in most countries, unsolicited proposals are allowed, but there are no particular rules for managing them. Yet, in some countries, public authorities are explicitly prevented from directly rewarding unsolicited project proposals; hence, the incentive to submit a project proposal can only come from the participation in the tender for its implementation, should the public authority decide to pursue the project. By contrast, other countries, such as Chile, Korea, Italy, and Taiwan, have adopted specific procedures for unsolicited proposals, which grant the project promoter an advantage at the implementation stage.

The prize vs contract rights issue is also central to public procurement of innovation. Europe is currently developing an explicit strategy of using public procurement to foster demand for innovative goods and services, but how contract rights should be assigned remains an open question. On the link between the R&D stage and the implementation stage, for instance, the European Commission has outlined two different procurement models. Under the "Pre-commercial procurement" (PCP) model, the public authority procures R&D activities, from the solution exploration phase to prototyping and testing, but it reserves the right to tender competitively the newly developed products or services.[1] By contrast, under the "Innovation Partnerships" model, development and production are procured through one single tender, with the innovator also obtaining the contract rights over the production of the innovation.[2]

In this paper, we study the interplay between *ex ante* incentives for R&D activities, potentially leading to valuable projects, and *ex post* efficiency in the implementation of the project. We consider a baseline model in which a *single* innovator can generate an idea by investing in costly research effort. Not only does this model reveal the gist of our insights clearly, but the single-innovator assumption is also quite realistic. In practice, many innovative projects procured by public agencies are unsolicited; and these proposals arrive one at a given point in time. The value of the idea is verifiable, and higher research effort leads (stochastically) to better ideas. The innovation gives rise to a project that can be implemented by a number of firms, including the innovator.

In this setting, the buyer has the dual objective of providing incentives for R&D effort *ex ante*, and implementing the project in the most effective way *ex post*. As the value of the proposal is verifiable, the innovator can be rewarded with a monetary prize as well as through the contract for the implementation of the project. We study how the optimal mechanism resolves the trade-off between R&D incentives at the innovation stage and allocative efficiency at the implementation stage.

As mentioned, monetary prizes have no impact on allocative efficiency, and as such may appear to be a superior way to provide incentives for innovation than contract rights. And indeed, prizes would be preferable in the absence of any agency problem at the implementation stage. The optimal mechanism would then reward the innovator with a monetary prize when its project is deemed by the buyer to be particularly worth pursuing, but the right to implement the project would be based purely on implementors' merits. The situation is different when agency problems generate some rents at the implementation stage, as these rents can then potentially be used to motivate the innovator, at no additional cost for the buyer. Indeed, we show that contract rights can be a central tool for providing incentives for innovation.

To study this issue, we consider the case where potential implementors are privately informed about their costs. We find that the optimal mechanism provides incentives for research effort through a combination of monetary prizes and implementation contract rights.

[1] See EC (2007) and http://cordis.europa.eu/fp7/ict/pcp/home_en.html

[2] See the EU (2014), the EU Directive 2014/24/EU, available at.http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32014L0024

The intuition is simple: A standard procurement auction would pick the implementor with the lowest cost but would also give it an informational rent; it is instead optimal to use this rent to incentivize the innovator. Hence, when the proposal is of sufficient merit, the innovator is favored in the implementation tender, and may thus win the contract even if its cost is not the lowest. Furthermore, the project may be implemented even when, because of high costs, it would not be implemented in a standard procurement auction. Conversely, the optimal mechanism biases the implementation tender against the innovator when the project has a low value. Monetary prizes, however, remain valuable: If the project is of particular merit, the innovator may receive a financial compensation in addition to being favored in the implementation tender. Finally, whenever such a prize is awarded, this is regardless of whether or not the innovator eventually wins the contract.

Comparative statics reflect the same insights. When firms' costs are more heterogeneous, informational rents become more significant, which makes it optimal for the buyer to rely more on contract rights to incentivize innovation. By contrast, the optimal mechanism involves a prize (that is, contract rights alone are not sufficient) when costs are not very heterogeneous or the number of firms is high (as the innovator's information rent is low), or when the value of innovation is high (as the information rent may be insufficient to motivate effort).

Equipped with these findings, we extend the model to allow for multiple innovators. This situation is relevant when the buyer has a clear sense about the type of innovation she needs and its feasibility. We show that the above insights carry over. First, our results confirm the optimality of the use of contract rights for rewarding innovations: The project values affect the optimal allocation of contract rights, with the proposer of a high-value project being favored at the implementation stage. Second, a "winner-takes-all" principle holds, in the sense that at most one project is ever awarded a prize; this monetary prize is now warranted when the project is particularly valuable and/or when an innovator's research effort is particularly worth being incentivized.

Our analysis also sheds some light on whether the selection of the project and the choice of the implementor should be independent or joint decisions. When the choice of the project does not affect implementors' informational rents, the project selection can be made independently of the choice of the implementor; a project is then chosen based solely on its merit, without regard to which firm will implement the project. Still, as in the single-innovator case, the choice of the implementor remains biased in favor or against the innovator, depending on the value of its proposal. When instead the choice of the project affects implementors' informational rents, it can be optimal to distort the project selection as well; in this case, both the project selection and the choice of the implementor should depend on the values of the proposals as well as on implementation costs.

The paper is organized as follows. In Section 2 we discuss the related literature. In Section 3 we study the case of a single innovator, which is relevant for unsolicited proposals. Section 3.1 sets up the model, Section 3.2 presents a number of benchmarks, and section 3.3 develops the main analysis. In Section 4 we extend the analysis to the case of with multiple innovators, which is more relevant for the procurement of innovation. In Section 5 we discuss the insights of our analysis for the approaches used in practice for unsolicited proposals and innovation procurement. In Section 6 we make some concluding remarks.

2 Related Literature

On prizes versus property rights to motivate innovation. Our issue of prize vs contract is reminiscent of the well-known debate about the patent systems as an effective method of motivating innovation – see Maurer and Scotchmer (2004) and Cabral *et al.* (2006) for reviews. Just as in our model, the patent system involves *ex post* distortion (both in terms of too little quantity and of foreclosure on competing firms), making prizes apparently preferable – see, e.g., Kremer, 1998. Yet the literature has shown that, as in this paper, *ex post* distortion can be an optimal way to motivate *ex ante* innovation. The difference lies in the motivation for the *ex post* distortion. In the case of Weyl and Tirole (2012), for instance, the supplier has private information at the *ex ante* (innovation) stage; he obtains property rights in order to facilitate information revelation. In our case it is the private information in the *ex post* stage which, coupled with limited liability, forces the buyer to give up rents. These rents can be used as incentives for innovation, by tilting the allocation of contract rights.

On bundling sequential tasks. Our analysis is related to the literature on whether two tasks should be allocated to the same agent ("bundling") or to two different agents ("unbundling"). A first literature has shown that this choice can be driven by problems of adverse selection (see, e.g., Ghatak, 1997; Armendariz and Gollier, 1998), monitoring (Besley and Coate, 1995; Armendariz, 1999; Rai and Sjöström, 2004), moral hazard (Stiglitz, 1990; Varian 1990; Holmstrom and Milgrom, 1991; Itoh, 1993), or agents' limited liability (Laffont and Rey, 2003). More recently, a second strand of literature has focused more specifically on sequential tasks, highlighting the role of externalities across tasks (Bennett and Iossa, 2006), budget constraints (Schmitz, 2013), information on the *ex post* value of the second task (Tamada and Tsai, 2007), or competition among agents (Li and Yu, 2011). Our paper contributes to this literature by showing that the decision on implementation should depend on the value of the proposed project(s) as well as on implementors' characteristics; full unbundling is therefore typically not optimal; also, pure bundling is optimal only under rather specific conditions – namely, when the innovator is in a much better position to implement its project.

On discrimination and bidding parity in auctions. Our analysis is also related to the literature on discrimination in auctions, which finds it optimal to distort the allocation to reduce the informational rents accruing to the bidders: Discrimination against efficient types helps levelling the playing field, and thus elicits more aggressive bids from the otherwise stronger bidders (Myerson, 1981, and McAfee and McMillan, 1985). In a similar vein, when bidders can invest in cost-reduction, an *ex post* bias in the auction design can help foster bidders' *ex ante* investment incentives (Bag 1997) or prevent the reinforcement of asymmetry among market participants (Arozamena and Cantillon, 2004). Likewise, manipulating the auction rules can help motivate selfish investment in cost reduction by an incumbent firm (Laffont and Tirole, 1988), or favor the adoption of an efficient technology by an inefficient firm (Branco, 2002). We contribute to this literature by showing that when investment is "cooperative" (in the sense of Che and Hausch, 1999) and directly benefits the buyer, both favoritism and handicapping are optimal, depending on the value of the proposed project and on bidders' costs.

3 Unsolicited Proposals

We consider here the case where a single innovator may come up with a project, as is often the case for unsolicited proposals. The decision facing the buyer is whether to adopt the project and if so, how to implement it via competition among multiple firms.

3.1 Model

A principal (buyer) oversees a project involving two stages: Innovation and Implementation.

In the first stage an innovator, say firm 1, invests in developing the project proposal: Exerting effort $e \geq 0$ costs $c(e)$ to the innovator, and leads to a project whose worth for the principal is a random variable v. The variable v is drawn from $V := [\underline{v}, \bar{v}]$ according to a c.d.f. $F(\cdot|e)$, which admits a differentiable density $f(\cdot|e)$ in the interior. We assume that raising e shifts the distribution $F(\cdot|e)$ in the sense of Monotone Likelihood Ratio Property:

$$(MLRP) \qquad \text{For any } v' > v \text{ and } e' > e, \qquad \frac{f(v'|e')}{f(v|e')} > \frac{f(v'|e)}{f(v|e)}.$$

Once the project proposal is unveiled, the value v becomes publicly observable and verifiable. Verifiability of v is a reasonable assumption in many procurement contexts, where the project the buyer wishes to procure is described in the tender documents through precise functional and performance terms. For example, the request for proposal (RFP) may specify technology improvements for faster medical tests, transport units with lower energy consumption, ICT systems with interoperability characteristics, and so on. In these cases, v may capture respectively a speed increase with the medical test, the degree of energy efficiency of the transport unit, the interoperability features of the ICT system, or other technical functionalities specified in the tender and verified in the submitted prototypes.

In the second stage, n potential firms, including the innovator, compete for implementing the project. Each firm $i \in N := \{1, ..., n\}$ faces a cost θ_i, which is privately observed and drawn from $\Theta := [\underline{\theta}, \overline{\theta}]$ according to a c.d.f. $G_i(\cdot)$, which admits density $g_i(\cdot)$ in the interior. We assume that $\underline{\theta} < \overline{v}$. If the project is not implemented, all parties obtain zero payoff. We assume that $G_i(\theta_i)/g_i(\theta_i)$ is nondecreasing in θ_i, for each $i \in N$.

Suppose that the innovator has made an effort e and generates an innovation worth v; if the principal pays t for the project, then the principal internalizes the welfare of

$$v - t.$$

From the Revelation Principle, without loss of generality we can restrict attention to direct revelation mechanisms. The timing of the game is thus as follows:

1. The principal offers a direct revelation mechanism specifying the allocation decision (i.e., whether the project will be implemented, and if so by which firm) and a payment to each firm, as functions of firms' reports on their costs.

2. The innovator chooses e; the value v is realized and observed by all parties.

3. Firms observe their costs and decide whether to participate.

4. Participating firms report their costs; the project is implemented (or not) and transfers are made according to the procedure.

We assume here that firms are free to opt in or out, upon learning their cost. This is a natural assumption in many settings, and particularly so in the case of unsolicited proposals, because until the nature of the project – its value and the costs of implementing it – is determined, the identities of candidates capable of carrying out the project will not be known. This makes it difficult for the principal to solicit the relevant firms and force them to buy in. This opt-out assumption implies that the principal needs to give up some information rents to the selected implementor.[3]

We also assume that the principal must at least break even for each realized value v of the project. This assumption, to be formalized more precisely later, could arise from the political feasibility constraint that a project that is likely to run a loss would be disapproved by either a legislative body or a project evaluation authority.[4]

3.2 Benchmarks

To facilitate the interpretation of the results that follow, it is useful to begin with a couple of benchmarks. In both benchmarks, we assume that a project worth v is available exogenously; i.e., no innovation is needed. Hence, the only problem facing the principal is to decide wether to implement the project and if so by which firm in N. The two benchmarks differ with regard to the information available on implementation costs.

Suppose first that the principal observes firms' implementation costs, and can thus have any firm i implement the project by simply paying its true cost θ_i. It is therefore optimal for the principal to select the firm with the lowest cost θ_i, provided that this cost does not exceed the project value v. Hence, in the first-best allocation, each firm i obtains the contract with probability:

$$x_i^{FB}(v,\theta) := \begin{cases} 1 & \text{if } \theta_i < \min\{v, \min_{j\neq i}\theta_j\}, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose now that the principal does not observe firms' implementation costs. From the revelation principle, the principal can again offer a direct mechanism $(x,t) : V \times \Theta^n \to \Delta^n \times \mathbb{R}^n$, where $\Delta^n := \{(x_1, ..., x_n) \in [0,1]^n | \sum_{i\in N} x_i \in [0,1]\}$, specifying the probability

[3] In the absence of this interim participation constraint, the principal could implement the project without any rents accruing to the firms, because the firms could be required to "buy-in" to a contract, via an upfront fee. As a result, the first best could be achieved at the implementation stage, and there would be no gain from using contract rights to reward the innovator; monetary prizes would indeed be preferable.

[4] Analytically, this assumption prevents the principal from using an incentive scheme that awards an arbitrarily high bonus to the innovator with a vanishing frequency. Such a scheme may be of theoretical interest but is unreasonable and unrealistic.

$x_i(v,\theta)$ that firm i implements the project and the transfer payment $t_i(v,\theta)$ that it receives, when firms report types $\theta := (\theta_1, ..., \theta_n)$. As firms can opt-out, the mechanism (x,t) must be *interim individually rational*:

$$\forall i \in N, \forall v \in V, \forall \theta_i \in \Theta, \qquad U_i(v,\theta_i) \geq 0, \tag{IR}$$

where $U_i(v,\theta_i)$ denotes firm i's *interim* expected profit:

$$U_i(v,\theta_i) := \mathbb{E}_{\theta_{-i}}\left[t_i(v,(\theta_i,\theta_{-i})) - \theta_i x_i(v,(\theta_i,\theta_{-i}))\right],$$

with $\theta_{-i} := (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$. The revelation principle further requires the mechanism to be *interim incentive compatible:*

$$\forall i \in N, \forall v \in V, \forall\left(\theta_i,\theta_i'\right) \in \Theta^2, \qquad U_i(v,\theta_i) \geq u_i(v,\theta_i'|\theta_i), \tag{IC}$$

where

$$u_i(v,\theta_i'|\theta_i) := \mathbb{E}_{\theta_{-i}}\left[t_i(v,(\theta_i',\theta_{-i})) - \theta_i x_i(v,(\theta_i',\theta_{-i}))\right]$$

denotes the *interim* expected profit that firm i could obtain by reporting a cost θ_i' when it actually faces a cost θ_i.

The problem facing the principal is then to solve

$$\begin{array}{cc} \max\limits_{x,t} & \mathbb{E}_\theta\left[w\left(v,\theta\right)\right], \\ & \text{subject to } (IR), (IC) \end{array}$$

where

$$w\left(v,\theta\right) := \sum_{i\in N}\left[x_i\left(v,\theta\right)v - t_i\left(v,\theta\right)\right]$$

denotes the principal's *ex post* surplus.

Note that we have ignored the constraint that the principal cannot make a loss, as this constraint will never be binding here.

This constitutes a standard optimal (procurement) auction problem, the solution of which is characterized as follows.

PROPOSITION 1. *(Myerson) The optimal procurement mechanism assigns the contract to firm i with probability:*

$$x_i^{SB}(v,\theta_i) := \begin{cases} 1 & \text{if } J_i(\theta_i) \leq \min\left\{v, \min_{j\neq i} J_j(\theta_j)\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ is firm i's virtual cost.

3.3 Optimal mechanism

We now consider our main problem in which the principal faces *ex post* adverse selection with respect to firms' implementation costs as well as *ex ante* moral hazard problem with respect to the innovator's effort. As in the standard auction problem just considered, the principal offers a direct mechanism $(x, t) : V \times \Theta \to \Delta^n \times \mathbb{R}^n$. As firms can opt out upon the realization of their costs, the mechanism must satisfy (IR). And as these costs are private information, the mechanism must also satisfy (IC). In addition, the mechanism cannot impose any loss on the principal, and must therefore satisfy a *limited liability* condition:

$$\forall v \in V, \quad \mathbb{E}_\theta \left[w\left(v, \theta\right) \right] \geq 0. \tag{LL}$$

Finally, as the innovator chooses effort e in its best interest, the mechanism must also satisfy the following *moral hazard* condition:

$$e \in \arg\max_{\tilde{e}} \left\{ \mathbb{E}_{v,\theta} \left[U_1(v, \theta_1) \mid \tilde{e} \right] - c(\tilde{e}) \right\}. \tag{MH}$$

The principal's problem is to choose an optimal mechanism satisfying these constraints. More formally, she solves the problem:

$$[P] \qquad \max_{x,t,e} \quad \mathbb{E}_{v,\theta} \left[w\left(v, \theta\right) \mid e \right], \quad \text{subject to } (IR), (IC), (LL), (MH).$$

Throughout the analysis, we assume that an optimal mechanism exists, which induces an interior effort level e^*. The following Proposition then characterizes this optimal mechanism:

Proposition 2. *There exists $\lambda \geq 0$ such that, in the optimal mechanism solving $[P]$, firm $i = 1, ..., n$:*

(*i*) *Obtains the contract with probability:*

$$x_i^*(v, \theta) = \begin{cases} 1 & \text{if } K_i(v, \theta_i) \leq \min\left\{v, \min_{j \neq i} K_j(v, \theta_j)\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$K_i(v, \theta_i) := \begin{cases} J_i\left(\theta_i\right) - \min\left\{\beta\left(v\right), 1\right\} \frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1 \\ J_i\left(\theta_i\right) & \text{if } i \neq 1 \end{cases}, \text{ with } \beta\left(v\right) := \lambda \frac{f_e(v|e^*)}{f(v|e^*)}.$$

(*ii*) *Earns a transfer:*

$$t_i^*(v, \theta_i) := \rho_i(v) + \int_{\theta_i}^{\overline{\theta}} X_i^*(v, s) ds,$$

where:

– $X_i^*(v, \theta_i)$ *denotes firm i's expected probability of obtaining the contract:*

$$X_i^*(v, \theta_i) = \mathbb{E}_{\theta_{-i}}[x_i^*(v, (\theta_i, \theta_{-i}))],$$

– $\rho_i^*(v) := 0$ *for* $i \neq 1$, *and*

$$\rho_1^*(v) := \begin{cases} \mathbb{E}_\theta\left[\sum_{i \in N} x_i^*(v, \theta)\left[v - J_i(\theta_i)\right]\right] > 0 & \text{if } \beta(v) > 1, \\ 0 & \text{if } \beta(v) < 1. \end{cases}$$

Proof. See Appendix A. □

To gain more intuition about this characterization, it is useful to decompose the principal's payment to each firm into two components. The first component is the information rent that she must pay to elicit the firm's private information. The usual revenue equivalence argument means that this component is uniquely tied to – and should thus be interpreted as being necessitated by – the awarding of contract to a firm. We thus call this the *contract payment*. The second component is the constant amount paid to all cost types of a firm, including the highest cost type $\overline{\theta}$. As this component is not warranted by information problems, we call it the *cash prize*, and denote it by $\rho_i^*(v)$. Normally, the principal would have no reason to award cash prizes to firms, and here this is the case as well for all non-innovating firms $i = 2, ..., n$. For the innovating firm $i = 1$, however, a cash prize may be necessary. The central question is how the principal should combine these two types of payment to encourage innovation.

To answer this question, suppose the principal pays a dollar to firm 1 for a project worth v. This contributes to the innovation incentive, in the sense of relaxing (MH), by $\frac{f_e(v|e)}{f(v|e)}$. Multiplied with the shadow value λ of relaxing (MH), $\beta(v) = \lambda \frac{f_e(v|e)}{f(v|e)}$ gives the *incentive benefit* to the principal of relaxing (MH).[5] If moral hazard is not a concern, which is the case if the innovator either has no control over v, or can choose the optimal effort at close to zero cost, then $\lambda = 0$, in which case $\beta(v) = 0$ so the optimal mechanism coincides with the standard second-best solution described in Proposition 1. If instead the moral hazard problem is not trivial, then the standard procurement auction is not optimal:

Lemma 1. *Whenever the standard procurement auction yields an interior solution in the effort level, then* $\lambda > 0$ *and the mechanism specified in Proposition 1 is not optimal.*

[5] The reader may recall that this term figures also prominently in the moral hazard literature, starting with Holmstrom (1979), determining the incentive provided the principal to the agent to motivate the latter's effort.

Proof. See Appendix B. □

Lemma 1 shows that the moral hazard constraint is binding whenever the standard procurement auction induces an interior level of effort. The simple intuition is that this procurement auction optimally extracts part of the benefit from the effort, leaving the innovator unable to capture the full benefit from her effort. Given that the procurer does not bear the cost of the effort, at the effort level the innovator finds optimal, the procurer prefers to induce a higher effort. The optimal research effort equates the social benefit from a higher expected value project, due to a better probability distribution $F(\cdot|e)$, with the marginal cost of that effort, $c'(e)$. The innovator instead equates this marginal cost to the marginal benefit from a greater expected information rent.[6] Yet, the moral hazard problem is not trivial because, absent explicit incentive distortions ($\lambda = 0$), the expected information rent is lower than the expected value of the project.

Given $\lambda > 0$, the incentive benefit β is nonzero and thus affects the selection of the implementor. For any firm other than the innovator (i.e., $i \neq 1$), the shadow cost $K_i(v, \theta_i)$ of awarding the contract to that firm is simply given by Myerson's usual virtual cost, $J_i(\theta_i)$, which accounts for both the production cost θ_i and the information rent $G_i(\theta_i)/g_i(\theta_i)$. For the innovator, however, the shadow cost differs from this virtual cost by the incentive benefit that awarding the contract to the innovator generates for the principal. This incentive benefit depends on the value v of the project: Intuitively, rewarding the innovator for a low-value project (an evidence for a low effort) hurts innovation incentives, whereas rewarding the firm for a high-value project (an evidence for a high effort) enhances its incentive for innovation.

The MLRP indeed implies that $\beta(v) = \lambda \frac{f_e(v|e)}{f(v|e)}$ increases in v, and there exists a unique $\tilde{v} \in (\underline{v}, \overline{v})$ such that $\beta(\tilde{v}) = 0$. Consider first a project worth $v < \tilde{v}$. Rewarding the innovator in this case reduces its innovation incentive: $\beta(v) < 0$. Hence, it is not optimal for the principal to award a cash prize to the innovator. For the same reason, each dollar paid as information rents harms the innovator's incentive, causing the shadow cost $K_1(v; \theta_1)$ of assigning the contract to the innovator to *exceed* its virtual cost $J_1(\theta_1)$, by $-\beta(v)\, G_i(\theta_i)/g_i(\theta_i)\,(> 0)$. Hence, the optimal mechanism calls for biasing the contract allocation against the innovator, in comparison with the second-best.

Consider next a project worth $v > \tilde{v}$. There are two possibilities. Suppose first $v < \hat{v} := \sup\{v \in V \mid \beta(v) \leq 1\}$. In this case, the incentive benefit $\beta(v)$ of paying a dollar to the innovator is positive but less than one. Hence, it is still optimal to award no cash prize,

[6] Note that the *ex post* information rent of the contractor, $G_1(\theta_1)/g_1(\theta_1)$, is invariant with the value of the project v, the expected information rent, $\mathbb{E}_{v,\theta}[x_1(v,\theta)G_1(\theta_1)/g_1(\theta_1)]$, increases with the contractor's effort e, as high-value projects are implemented more often (from Proposition 2).

as this would entail a net loss for the principal. However, a fraction $\beta(v)$ of the information rent accruing to the innovator goes toward its innovation incentive, which *reduces* the shadow cost $K_1(v, \theta_1)$ of assigning contract to the innovator below its virtual cost $J_1(\theta_1)$, by $\beta(v) G_i(\theta_i)/g_i(\theta_i)$. Hence, compared with the second-best benchmark, the optimal mechanism distorts allocation of contract in favor of the innovator.

Finally, suppose $v > \hat{v}$. In this case, a dollar payment to the firm yields more than a dollar incentive benefit. A cash prize is then clearly beneficial, which is why $\rho_1^*(v) > 0$. Hence, it pays the principal to transfer any surplus she collects, either through the cash prize or through the information rent; that is, (LL) is binding. Any increase in information rents for the innovator simply crowds out the cash prize by an equal amount. Hence, the entire information rent paid to the innovator is justified on the incentive ground but, due to this crowding-out effect, its incentive benefit is only 1 (and not $\beta(v) > 1$). Therefore, the shadow cost $K_1(v, \theta_1)$ reduces to the production cost θ_1. Compared with the second-best, the optimal mechanism distorts allocation of contract in favor of the innovator, to such an extent that the innovator is treated as an "in-house" supplier. Any further distortion in favor of the innovator reduces the total "pie" – and thus the cash prize to the innovator – more than it increases the information rent to that firm, and is thus suboptimal.

We state these observations more formally as follows:

Corollary 1. *There exists $\tilde{v}$ and $\hat{v}$, with $\underline{v} < \tilde{v} < \hat{v} \leq \overline{v}$, such that the optimal mechanism has the following characteristics:*

- *If $v < \tilde{v}$, then no prize is awarded, and $x_1^*(v, \theta) \leq x_1^{SB}(v, \theta)$ whereas $x_i^*(v, \theta) \geq x_i^{SB}(v, \theta)$ for all $i \neq 1$;*
- *If $\tilde{v} < v < \hat{v}$, then no prize is awarded, but $x_1^*(v, \theta) \geq x_1^{SB}(v, \theta)$ whereas $x_i^*(v, \theta) \leq x_i^{SB}(v, \theta)$ for all $i \neq 1$;*
- *If $v > \hat{v}$, then a prize is awarded to the innovator and $x_1^*(v, \theta) \geq x_1^{SB}(v, \theta)$, whereas $x_i^*(v, \theta) \leq x_i^{SB}(v, \theta)$ for all $i \neq 1$.*

Whether it is optimal to award a monetary prize (i.e., $\hat{v} < \bar{v}$) depends on how much efforts incentives need to be given, and on how much of them would already have been provided by the informational rents stemming from a standard second-best auction. We show for instance in Appendix C that awarding a prize can be optimal when the set of possible values is large (as innovation incentives then matter a lot) as well as when there is either little cost heterogeneity, or a large number of firms (in which case the procurement auction does not generate much informational rents).

Corollary 1 shows that the optimal mechanism departs from a standard second-best auction in different ways for high-value and low-value projects. This mechanism can moreover be easily implemented in practice, using simple variants to common procurement designs.

- $v > \tilde{v}$: *Bonus.* In this range, the contract allocation is biased in favor of the innovator, so the innovator may implement the project despite not being the most efficient firm. In practice, this could be achieved by giving the innovator a bidding credit in the tendering procedure. Bidding credits can take many forms, but most commonly they consist of additional points in the score of the original proponent's bid or of financial support for bidding purposes. This system is for example adopted in Chile and Korea.
- $v < \tilde{v}$: *Handicap.* In this range, the contract allocation is biased against the innovator, who may not implement the project despite being the most efficient firm. We are not aware of the use of such bias for procuring innovative projects; such handicap systems are however used for instance when governments want to favor own domestic industry's supplies.[7] We discuss this further below (see the remark on handicaps).

We note further that in the region where a monetary prize is optimal the mechanism can be implemented in a very familiar and simple manner:

- $v > \hat{v}$: *Full delegation.* In this region, the innovator is awarded a monetary prize $\rho_1^*(v)$, equal to the full value of the project (net of informational rents), and is allocated the contract if $\theta_1 < \min\{v, \min_{i\neq 1} J_i(\theta_i)\}$. This can be achieved by delegating the procurement to the innovator, for a fixed price equal to the value of the project. Indeed, suppose the principal offers a payment v to the innovator to deliver the project either by itself or through subcontracting with a different firm. The innovator then acts as a prime contractor with the authority to assign production. Facing the price $v > \hat{v}$, and given θ_1, the innovator chooses $(x(v,\cdot), t(v,\cdot)) : \Theta^n \to \Delta \times \mathbb{R}^{n-1}$ so as to solve:

$$\max_{x,t} \; \mathbb{E}_{\theta_{-1}} \left[(v-\theta_1)x_1(v,\theta_1,\theta_{-1}) + \textstyle\sum_{i\neq 1} \{v x_i(v,\theta_1,\theta_{-1}) - t_i(v,\theta_1,\theta_{-1})\} \right], \quad \text{subject to } (IR) \text{ and } (IC)\,.$$

[7] Under "preferential price margin", purchasing entities accept bids of domestic suppliers over foreign suppliers as long as the difference in price does not exceed a specific margin of preference. The price preference margin can result from an explicit "buy local policy," e.g., "Buy America Act."

The standard procedure of using the envelope theorem and changing the order of integration results in the optimal allocation x solving:

$$\max_{x,t} \mathbb{E}_{\theta_{-1}} \left[(v - \theta_1) x_1(v, \theta_1, \theta_{-1}) + \sum_{i \neq 1} [v - J_i(\theta_i)] \, x_i(v, \theta_1, \theta_{-1}) \right],$$

or implementing the optimal allocation x^* for the case of $v > \hat{v}$.

The above results have also implications for whether or not to implement a project. For instance, for $n = 1$ we have:

- For $\underline{v} < v < \tilde{v}$, $K(v, \theta) > J(\theta)$ $(> \theta)$: Compared with the first-best, there is a downward distortion – under-implementation of the project – which is even more severe than in the standard second-best.
- For $\tilde{v} < v < \hat{v}$, $J(\theta) < K(v, \theta) < \theta$: There is still a downward distortion compared with the first-best, but it is less severe than in the standard second-best.
- For $v \geq \hat{v}$, we have $J(\theta) < K(v, \theta) = \theta$: There is no distortion anymore; the project is implemented whenever it should be, from a first-best standpoint.

Remark: On the feasibility of handicaps. The optimal mechanism relies on a "stick and carrot" approach: It rewards good proposals by conferring an advantage in the procurement auction (possibly together with a monetary prize) and punishes instead weak proposals with a handicap in the procurement auction. Yet in practice, whilst many innovation procurement mechanisms involve innovation prizes or distort the contract allocation in favor of the innovators, handicaps for weak projects do not appear to be used. This may stem from the risk of manipulation: An innovator with a low-value project may for instance strategically choose to participate in the implementation tender through a different firm, in order to avoid the handicap.

To see how the mechanism would need to be adjusted if handicaps were explicitly ruled out, suppose that the innovator cannot be left worse off than under the standard second best allocation.[8] That is, the mechanism must take into account the additional constraint:

$$x_1(v, \theta) > x_1^{SB}(v, \theta).$$

[8] It can be checked that this indeed ensures that the innovator is never worse off than a pure contractor – see Online Appendix A.

In the Online Appendix, we show that, keeping constant the multiplier λ of the innovator's incentive constraint, ruling out handicaps has no impact on the contract right for high-value projects (namely, those with $v \geq \tilde{v}$), as $x_1^*(v,\theta) > x_1^{SB}(v,\theta)$ in this case. By contrast, for low-value projects (i.e., those with $v < \tilde{v}$), the no-handicap constraint is binding and the contract right increases from $x_1^*(v,\theta)$ to $x_1^{SB}(v,\theta)$. Interestingly, the no-handicap constraint does not affect the size of the prize. Of course, removing the "stick" raises the cost of providing innovation incentives, and thus we would expect an increase in the multiplier of the incentive constraint λ (implying that the favorable bias for a high-value project is larger, and the monetary prize more often awarded) and a reduction in the optimal innovation effort.

Illustration. To illustrate the above insights, consider the following example: (i) Implementation costs are uniformly distributed over $\Theta = [0,1]$; (ii) the innovator can exert an effort $e \in [0,1]$ at cost $c(e) = \gamma e$; and (iii) the value v distributed on $[0,1]$, according to the density $f(v|e) = e + (1-e)\,2\,(1-v)$; that is, exerting effort increases values in the MLRP sense, from a triangular density peaked at $v = 0$ for $e = 0$ (where in particular $f(1|0) = 0$), to a better (uniform, actually) distribution for $e = 1$. Note that $f_e(v|e) = 2v - 1 \gtreqless 0 \Longleftrightarrow v \gtreqless \tilde{v} = 1/2$.

The linearity of the cost and benefits ensures that it is optimal to induce maximal effort ($e^* = 1$) as long as the unit cost γ is not too high. Conversely, as long as $e^* = 1$, the Lagrangian multiplier λ increases with the cost γ.

Consider first the case where only the innovator can implement its project (i.e., $n = 1$). Figure 1 depicts the associated expected probability $p^*(v) := \mathbb{E}_\theta[x_1^*(v,\theta)]$ that the project is implemented, as a function of its value v, in the standard second-best and in the optimal mechanism, for different values of the Lagrangian multiplier λ (or, equivalently, to different values of γ inducing $e^* = 1$).

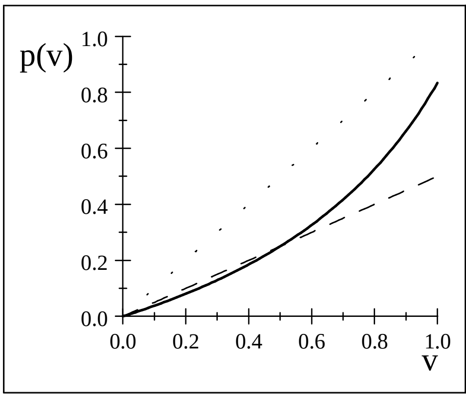

Figure 1a: λ "small"

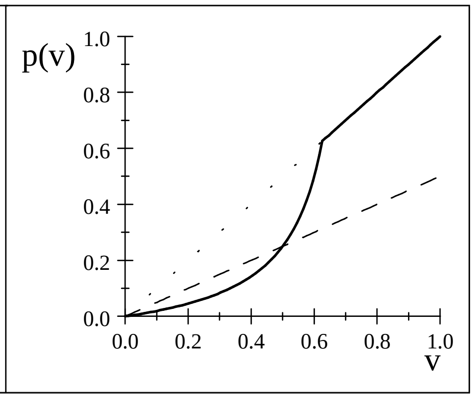

Figure 1b: λ large

In the first-best $p^{FB}(v) = \Pr[\theta < v] = v$ whereas in the standard second best, which is obtained for $\lambda = 0$, $p^{SB}(v) = \Pr[J(\theta) = 2\theta < v] = v/2$. For $\lambda > 0$ but "small" (see Figure 1a, where $\lambda = 0.8$), $p^*(v) = \Pr[K(v,\theta) < v]$, where $K(v,\theta) > J(\theta)$, and thus $p(v) < p^{SB}(v)$, for $v < \tilde{v} = 1/2$, whereas $K(v,\theta) < J(\theta)$, and thus $p(v) > p^{SB}(v)$, for $v > 1/2$; and for λ large enough (see Figure 1b, where $\lambda = 4$), there exists a range $v > \hat{v}\,(> 1/2)$ where $K(v,\theta) = \theta$ and thus $p(v) = p^{FB}(\theta)$.

Consider now the case where a second firm can implement the project as well (i.e., $n = 2$). Figure 2 depicts for instance the outcome of the implementation tender for the case $n = 2$ (where firm 2's cost is also uniformly distributed over $\Theta = [0,1]$):

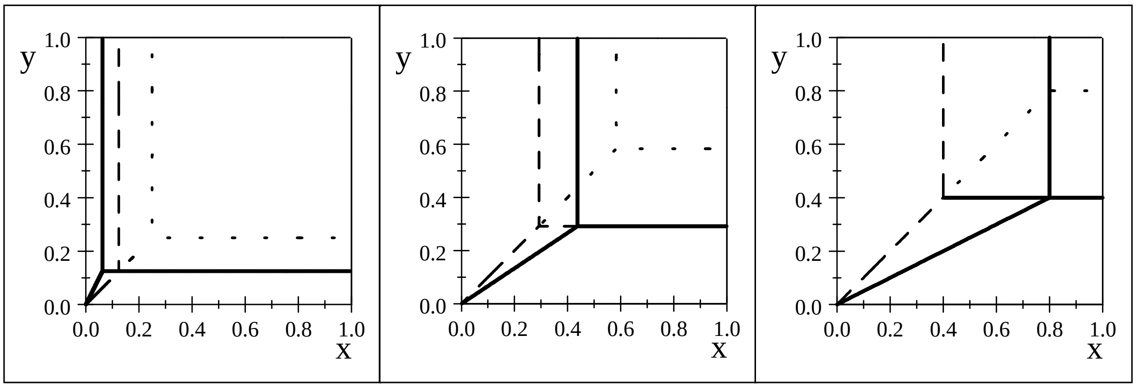

Figure 2a: $v < \tilde{v}$. Figure 2b: $\tilde{v} < v < \hat{v}$. Figure 2c: $v > \hat{v}$

- For $\underline{v} < v < \tilde{v}$ (see Figure 2a, where $\lambda = 4$ and $v = 1/4$): Compared with the first-best or the standard second-best, where the more efficient implementor would be selected, it is now optimal to bias the allocation of the contract against the innovator; in particular, the innovator obtains the contract less often, and the other contractor more often, than in the standard second-best. The project is implemented less often than in the second-best (and thus, *a fortiori*, than in the first-best).

- For $\tilde{v} < v < \hat{v}$ (see Figure 2b, where $\lambda = 4$ and $v = 7/12$): Compared with the first-best or the standard second-best, it is now optimal to bias the allocation of the contract in favour of the innovator, at the expense of the other implementor; as a result, the project is also implemented more often than in the standard second-best.

- Finally, for $v \geq \hat{v}$ (see Figure 2c, where $\lambda = 4$ and $v = 0.8$), the innovator's shadow cost corresponds to its actual cost; the allocation of the contract thus favours even more the innovator over the other firm; the project is also implemented substantially more often than in the standard second-best (in particular, it is now implemented whenever $\theta_1 < c$), but remains however implemented less often than in the first-best (e.g., when $\theta_2 < v < \theta_1, J(\theta_2) = 2\theta_2$).

4 Procuring innovation from multiple suppliers

We now assume that several firms may innovate and propose projects, as well as implement them. This case captures the problem of a buyer who has a specific need that readily available products or services cannot satisfy, and who decides to procure an innovation to satisfy this need. Illustrative examples include the Norwegian Department of Energy procuring a new technology for carbon capture and storage,[9] or the Scottish Government procuring low-cost, safe and effective methods of locating, securing and protecting, electrical array cables in Scottish sea conditions;[10] in both instances, the public authority put out a RFP, and multiple firms responded with different projects.

For the sake of exposition, we will suppose from now on that each firm $k = 1, ..., n$ can come up with a project of value v^k, which is publicly observable and distributed over V according to a c.d.f. $F^k(v^k|e^k)$ with density $f^k(v^k|e^k)$, where e^k denotes firm k's innovation effort.[11] We assume that firms decide on these efforts simultaneously, and denote by $\mathbf{e} = (e^1, ..., e^n)$ the profile of efforts. The alternative projects correspond to competing ways to fulfill a common social need, so they are perfect substitutes, meaning that the planner will choose one project at most. The previous setting corresponds to the special case where F^k is concentrated on $\underline{v}$ for all $k \neq 1$.

As there are now multiple innovators as well as multiple implementors, it is worth extending the analysis so as to allow for various types of interdependency between the projects and the implementors' costs. We thus assume that, firm i's cost of implementing project k is given by $\theta_i + \psi_i^k$, where:

- as before, θ_i is an idiosyncratic shock, privately observed by firm i, and distributed according the c.d.f. G_i;
- ψ_i^k represents an additional cost, potentially both project- and firm-specific, which for simplicity is supposed to be common knowledge.

Without loss of generality, we consider a direct revelation mechanism that specifies an allocation (namely, which project is implemented, if any, and if so by which firm) and a payment to each firm, as a function of realized project values, $\mathbf{v} = (v^1, ..., v^n)$, and of

[9] http://ted.europa.eu/udl?uri=TED:NOTICE:214787-2011:TEXT:EN:HTML&src=0

[10] http://ted.europa.eu/udl?uri=TED:NOTICE:436615-2013:TEXT:EN:HTML&src=0

[11] While formally all implementors are also innovators, the case of "pure contractors" can be accommodated by setting the density to zero for $v > \underline{v}$.

reported costs; a mechanism is thus of the form $(x,t): V^n \times \Theta^n \rightarrow \Delta^{n^2} \times \mathbb{R}^n$. The problem facing the principal can therefore be expressed as:

$$\max_{x,t,\mathbf{e}} \mathbb{E}_{\mathbf{v},\theta}\left[w\left(\mathbf{v},\theta\right) \mid \mathbf{e}\right],$$

where the *ex post* net surplus is now equal to

$$w\left(\mathbf{v},\theta\right) = \sum_{k,i \in N} \left[v^k x_i^k\left(\mathbf{v},\theta\right) - t_i\left(\mathbf{v},\theta\right)\right],$$

subject to (IR) and (IC), where now firms' *interim* expected profits when lying and reporting the truth are given by

$$u_i(\mathbf{v},\theta_i'|\theta_i) = \mathbb{E}_{\theta_{-i}}[t_i(\mathbf{v},\theta_i',\theta_{-i}) - \sum_{k\in N}(\theta_i + \psi_i^k)x_i^k(\mathbf{v},\theta_i',\theta_{-i})] \text{ and } U_i(\mathbf{v},\theta_i) = u_i(\mathbf{v},\theta_i|\theta_i),$$

and to the limited liability and moral hazard constraints, which respectively become:

$$\forall \mathbf{v} \in V^n, \qquad \mathbb{E}_\theta\left[w\left(\mathbf{v},\theta\right) \mid \mathbf{e}\right] \geq 0.$$

$$\forall i \in N, \qquad e^i \in \arg\max_{\tilde{e}^i} \mathbb{E}_{\mathbf{v},\theta}\left[U(\mathbf{v},\theta_i) \mid \tilde{e}^i, e^{-i}\right] - c_i(\tilde{e}^i).$$

The following Proposition provides a partial characterization of the optimal mechanism:

Proposition 3. *There exists a profile of efforts* $\mathbf{e}^* \geq 0$ *and* $\boldsymbol{\lambda} = \left(\lambda^1,...,\lambda^n\right) \geq 0$ *such that the optimal mechanism solving* $[P]$ *induces* $\mathbf{e}^*$*, and firm* $i = 1,...,n$*:*

- *Obtains the contract with probability:*

$$x_i^{k*}(\mathbf{v},\theta) = \begin{cases} 1 & \text{if } v^k - K_i(\mathbf{v},\theta_i) - \psi_i^k \geq \max\left\{0, \max_{(l,j)\neq(k,i)} v^l - K_j(\mathbf{v},\theta_j) - \psi_j^l\right\}, \\ 0 & \text{otherwise,} \end{cases}$$

  where

$$K_i(\mathbf{v},\theta_i) := J_i(\theta_i) - \left(\frac{\beta^i(v^i)}{\max\{\max_k \beta^k(v^k),1\}}\right)\left(\frac{G_i(\theta_i)}{g_i(\theta_i)}\right), \text{ and } \beta^i(v^i) := \lambda^i \frac{f_e^i(v^i|e^{i*})}{f^i(v^i|e^{i*})}.$$

- *Earns a transfer:*

$$t_i^*(\mathbf{v},\theta) := \rho_i^*(\mathbf{v}) + \sum_{k\in N}\psi_i^k X_i^{k*}(\mathbf{v},\theta_i) + \int_{\theta_i}^{\overline{\theta}} \sum_{k\in N} X_i^{k*}(\mathbf{v},s)ds,$$

  where firm i's interim probability of obtaining a contract under project k is now given by

$$X_i^{k*}\left(\mathbf{v},\theta_i\right) = \mathbb{E}_{\theta_{-i}}\left[x_i^{k*}\left(\mathbf{v},\theta_i,\theta_{-i}\right)\right]$$

whereas the monetary prize is positive only if $\beta^i(v^i) > \max\left\{\max_{j\in N}\beta^j(v^j), 1\right\}$*, in which case it is equal to:*

$$\rho_i^*(\mathbf{v}) := \mathbb{E}_\theta\left[\sum_{k,i\in N} x_i^{k*}(\mathbf{v},\theta)\left\{v^k - \psi_i^k - J_i(\theta_i)\right\}\right] > 0.$$

Proof. See Appendix D. □

To interpret this characterization, consider first the case where implementation known cost differences are additively separable across implementors and projects: $\psi_i^k = \psi_i + \psi^k$ for all i and k. The project selection can be made before awarding the implementation contract, in the sense that the former does not depend on the realized profile of θ_i's. Then the project selection is simply based on the "net values" of the projects, $v^k - \psi^k$, without regard to whom gets assigned the contract to implement the chosen project;[12] there is thus no need to wait until the realization of the costs before selecting the project. The project values however still affect the choice of the implementor, through their impact on virtual costs – the $K_i(\mathbf{v},\theta_i)$'s, which depend on all realized values, including those of unselected projects.[13] In particular, an increase in v^k raises both the probability that project k is selected, and the probability that firm k is chosen to implement it; in this sense, there is again a bias in favor of a contract with high-value innovation.

Outside the particular case mentioned above, the choices of the project and of the implementor are more intimately connected. Suppose for instance that $\psi_k^k = 0 < \psi_i^k = \bar{\psi}$ for all k and $i \neq k$. In this case, each firm has a cost advantage of $\bar{\psi}$ on the project it proposes, vis-à-vis other firms. For instance, imagine that there are two firms, and $v^1 > v^2$. If θ_2 is significantly lower than θ_1, the desire to exploit this cost advantage may lead the principal to choose project 2, because it comes with the additional cost advantage of $\bar{\psi}$.

A few other observations are worth making. First, as intuition suggests, the optimal allocation $x_i^{k*}(\mathbf{v},\theta)$ is nondecreasing in (v^i,θ_{-i}) and nonincreasing in $(\mathbf{v}^{-i,k},\theta_i)$. In addition, as all firms are now potential innovators, each virtual cost $K_i(\mathbf{v},\theta_i)$ is characterized by two cutoffs, $\tilde{v}^i$ and $\hat{v}^i$, defined as in the previous section, but with somewhat different implications. As before, each innovator benefits from a bias at the implementation stage when $v^i > \tilde{v}^i := \beta_i^{-1}(0)$, and is instead handicapped when $v^i < \tilde{v}^i$. How much the firm will actual benefit from

[12] To see this, note that the difference in surplus when a contractor i implements project k or project l is given by

$$\left(v^k - K_i - \psi^k - \psi_i\right) - \left(v^l - K_i - \psi^l - \psi_i\right) = \left(v^k - \psi^k\right) - \left(v^l - \psi^l\right),$$

and thus does not depend on which contractor i is selected.

[13] Note that for a "pure contractor," $K_i(\mathbf{v},\theta_i) = J_i(\theta_i)$, as in a standard second-best auction.

this bias, or be harmed from the handicap, now however depends on the relative performance of its project, and thus depends also on the values brought by the other projects, $\mathbf{v}^{-i}$.

Second, a "winner-takes-all" principle applies, in the sense that generically at most one firm is awarded a prize. As in the case of one innovator, a prize is worth giving only when the incentive benefit $\beta^i(v^i)$ exceeds one. But with multiple innovators, this may now happen to more than one firm at the same time, and there is therefore an additional effect to consider. Due to the limited liability of the buyer, an additional dollar paid to a firm, is a dollar less available to reward another firm. As the incentive benefit of a dollar is proportional to $\beta^i(v^i)$, the marginal benefit of the prize is maximized by attributing the prize to firm $\hat{\imath} = \arg\max_{i \in N} \{\beta^i(v^i)\}$. Splitting the available cash across firms is never optimal, for the same reason that it was never optimal to give less than the maximal prize to the innovator in the previous, single innovator case. In the same vein, even if $\beta^i(v^i) > 1$ for several firms, only firm $\hat{\imath}$ (for which this variable is the largest) will face undistorted virtual cost $K_{\hat{\imath}}(\mathbf{v}, \theta_{\hat{\imath}}) = \theta_{\hat{\imath}}$; the others will instead face a distorted virtual cost, equal to

$$K_i(\mathbf{v}, \theta_i) = \theta_i + \left[1 - \frac{\beta^i(v^i)}{\beta^{\hat{\imath}}(v^{\hat{\imath}})}\right] \frac{G_i(\theta_i)}{g_i(\theta_i)} > \theta_i.$$

Finally, note that if firms are *ex ante* symmetric (i.e., $f^k(.) = f(.)$ and $\psi_i^k = \psi$), then from MLRP the highest $\beta^i(v^i)$ corresponds to the highest v^i; hence, the best project is selected, and only that project can ever be awarded a prize. By contrast, if firms are not *ex ante* symmetric, then the prize will instead be given to the firm whose effort was most worth incentivizing (i.e., the firm with the highest $\beta^i(v^i)$), even if it is not the one with the best project (i.e., the highest v^i).

5 Discussion

5.1 Unsolicited Proposals

As mentioned in the Introduction, some countries do not allow public authorities to directly reward unsolicited proposals. Our analysis suggests instead that it can be optimal to reward valuable proposals through contract rights, and possibly by monetary prizes. Hodges and Dellacha (2007) describe three alternative ways used in practice:

∘ Bonus System. The system gives the original project proponent a bonus in the tendering procedure. This bonus can take many forms, but most commonly involves additional points in the score of the original proponent's technical or financial offer. This system is for example adopted in Chile and Korea. The original proponent first submits a proposal, which is not just an idea but typically contains a detailed description of the project and an impact study, and thus involves significant investment costs. The proposal is then reviewed by the appropriate governmental agency or ministry, who credits a bonus to the original proponent based on the project value. For example, in Chile the bonus credited to the first two unsolicited proposals for airport concessions obtained a bonus equal to 20 percent of the allowed score, whilst the third airport proposal received only 10 per cent.

∘ Swiss challenge system. The Swiss challenge system gives the original proponent the right to counter-match any better offers. It is most common in the Philippines and also used in Guam, India, Italy, and Taiwan.

∘ Best and final offer system. Here the key element is multiple rounds of tendering, in which the original proponent is given the advantage of automatically participating in the final round. It is used in Argentina, South Africa.

Our analysis suggests that these mechanisms have some merit, as biasing the implementation stage in favor of the innovator may indeed promote innovation. The Bonus System has also the additional merit of allowing the advantage to be linked to the value of the proposed project, with higher project values resulting in greater advantages. By contrast, the Swiss challenge system and the best and final offer system grant an unconditional advantage to the innovator, which in our analysis is sub-optimal. Note that none of these systems provides for an explicit handicapping.

5.2 Bundling R&D and Implementation

In the practice of innovation procurement, we observe two polar cases.

First, there is *unbundling,* where project selection and implementation are entirely kept separate; therefore the firm whose project was selected is treated exactly the same way as any other firm at the implementation stage. Examples include research contests, or the European Pre-commercial Procurement (PCP) model. In both cases, firms compete for innovative solutions at the innovation stage, and the best solution(s) may receive a prize. The procurer does not commit itself to acquire the resulting innovations.

Second, there is *pure bundling,* where the firm whose project is selected also implements it. This approach was for instance followed in US Defence Procurement in the 1980s, where the winner of the technical competition for the best prototype was virtually assured of being awarded the follow-on defence contract (see Lichtenberg, 1990; and Rogerson, 1994).

More recently, the European Procurement Directive 2014/24/EU has introduced the so-called "Innovation Partnerships" for the joint procurement of R&D services and large-scale production.

The previous section identifies circumstances (e.g., when $\psi_i^k = \psi_i + \psi^k$) where the project selection can be decoupled from cost considerations, in the sense that the project can be selected without knowing the costs of the implementors.[14] This may be relevant in practice when the cost advantage of the firm that submitted the project is low, as is the case for instance when a prototype conveys most of the required knowledge for the product production, e.g., because the innovation is either in the product design, or in the product mechanisms and materials. However, the above results also reveal that *unbundling* is never optimal: Even when the selection of the project can be decoupled from cost considerations, contract rights should still be used to reward innovation – i.e., the choice of the implementor should depend on the values of the projects (even of those which are not selected), as well as on reported costs.

By contrast, *pure bundling* can be optimal when there are great economies of scope between R&D and implementation (as when $\psi_k^k = 0 < \psi_i^k = \bar{\psi}$ for $i \neq k$, and $\bar{\psi}$ is large). This can be the case in practice for the procurement of complex IT systems, where the knowledge advantage of the software developer typically translates into a considerable cost advantage on the management and upgrading of the software. In this case, selecting the same firm for both R&D and implementation is likely to be better. However, even in that case, our results stress that the project selection should be based on both value and cost considerations; hence, the procurer should take cost considerations into account when selecting the project.

Whether or not to separate project implementation from project selection becomes a key question when Governments have a specific policy concern towards targeted groups such as *SMEs*. In both Europe and the US, procurement programs aimed at stimulating R&D investment from *SMEs* provide for separation between the R&D stage and the implementation stage, with funding provided based on firms's project proposals. The Small Business Innovation Research (SBIR) in the US or the UK Small Business Research Initiative (SBRI) are characterized by this separation between project selection and implementation.[15] Our analysis supports such approach. Indeed, *SMEs* may be at a clear disadvantage when the R&D competition is bundled with the contract implementation, as they may be unable to compete

[14] Such decoupling does not mean unbundling, however, for the selection of contractors still depends on the values of the proposals, as emphasized below.

[15] See respectively http://www.sbir.gov/ and https://sbri.innovateuk.org.

on large implementation contracts. Imagine in our framework that the government wishes to promote research effort specifically from *SMEs* and thus bans non-*SMEs* from proposing a project (as under SBIR and SBRI), and, to reflect the handicap as implementors, let us further assume that $\psi_i^k \to \Psi$ for all $i \in S$, where S is the set of *SMEs*, whereas $\psi_i^k = 0$ for $i \in C = N \backslash S$, the complementary set of Non-*SMEs*. The promotion of R&D effort only by *SMEs* implies $\lambda_i = 0$ for all $i \in C$: therefore our analysis shows that only *SMEs* may receive a prize, and in particular,

$$\forall i \in S, \rho_i^*(\mathbf{v}) > 0 \text{ if } \beta^i(v^i) > \max \left\{ \max_{j \in S} \beta^j(v^j), 1 \right\}.$$

Further, at the implementation stage, the *SMEs* are selected only if their additional cost Ψ more than compensates for the incentive value of contract rights; that is, whenever project k is selected, we have:

$$\forall i \in S, x_i^{k*}(\mathbf{v}, \theta) = \begin{cases} 1 \quad \text{if } K_i(\mathbf{v}, \theta_i) + \Psi \leq \min \left\{ \begin{array}{c} \min_{j \in S \backslash \{i\}} K_j(\mathbf{v}, \theta_j) + \Psi, \\ \min_{j \in C} J_j(\mathbf{v}, \theta_j), \\ v^k. \end{array} \right\}, \\ 0 \qquad \text{otherwise.} \end{cases}$$

SMEs are less likely to implement the selected project, the higher is Ψ and the smaller the policy concern towards promoting SMSs innovative effort (i.e. the smaller is λ_i). In the extreme case where $\Psi \to \infty$, our analysis shows that the mechanism should provide for the *SMEs* to compete for a prize but not be awarded contract rights.

A similar reasoning suggests that when base university research may play a key role in R&D activities, separation between selection and implementation may also help promoting their participation.

6 Conclusions

Procuring innovative projects requires incentivizing potential innovators' research efforts as well as an efficient implementation of the selected projects. Our analysis highlights a trade-off between these two objectives when implementors have private information about their costs. To solve this trade-off, the optimal mechanism relies on contract rights (possibly combined with monetary prizes).

A number of issues are worth exploring further. First, we have focused on situations where the value of the proposals can be contracted upon. This is a plausible assumption when for instance the proposal involves a prototype, or when performance measures – operational or productivity indicators, energy consumptions, emissions, etc. – are available and can be used in the tender documents; yet another possibility is to rely on evaluation committees. In other situations (e.g., base research), however, the difficulty of describing the project and/or non-verifiability issues may make it impossible to contract *ex ante* on the *ex post* values of the projects. Extending the analysis to these situations is beyond the scope of this paper but clearly constitutes an interesting avenue for future research.

Second, we have ignored the costs of participating to procurement tenders. In practice, submitting a tender bid may require tender development costs (e.g., complex estimations and legal advice) that involve significant economic resources, in which case biasing the tender in favor of the innovator may discourage potential implementors from participating in the tender. It would therefore be worth endogenizing the participation to the tender and exploring how the optimal mechanism should be adjusted to account for these development costs.

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Appendix

A Proof of Proposition 2

To solve $[P]$, we first reformulate (IC) in terms of interim allocation and payment rules. For each $i \in N$, and for any $v \in V$ and any $\theta_i \in \Theta_i$, let $X_i(v,\theta_i) := \int_{\theta_{-i}} x_i(v,\theta)\, dG_{-i}(\theta_{-i})$ and $T_i(v,\theta_i) := \int_{\theta_{-i}} t_i(v,\theta)\, dG_{-i}(\theta_{-i})$ denote the interim allocation and payment for firm i, and

$$U_i(v,\theta_i) := T_i(v,\theta_i) - \theta_i X_i(v,\theta_i) \tag{1}$$

denote firm i's expected profit. For each $i \in N$, (IC) then can be stated as

$$T_i(v,\theta_i) - \theta_i X_i(v,\theta_i) \geq T_i(v,\theta_i') - \theta_i X_i(v,\theta_i'), \forall v, \theta_i, \theta_i'.$$

The associated envelope condition then yields

$$U_i(v,\theta_i) = \rho_i(v) + \int_{\theta_i}^{\overline{\theta}} X_i(v,\theta) d\theta, \tag{2}$$

where

$$\rho_i(v) := U_i(v,\overline{\theta})$$

is the rent enjoyed by firm i when its cost is highest. Using (2), we can express firm i's expected rent as

$$\begin{aligned}\int_{\theta_i} U_i(v,\theta_i) d\theta_i &= \int_{\theta_i} \left[\rho_i(v) + \int_{\theta_i}^{\overline{\theta}} X_i(v,s) ds\right] dG_i(\theta_i) \\ &= \rho_i(v) + \int_{\theta_i} X_i(v,\theta_i) \frac{G_i(\theta_i)}{g_i(\theta_i)} dG_i(\theta_i). \end{aligned} \tag{3}$$

For each $i \neq 1$, the rent $\rho_i(v)$ does not help relax any constraint and reduces the surplus for the principal, so it is optimal to set $\rho_i(v) = 0$ for all v.

Using (1) and (3), the total expected transfer to the firms can be expressed as:

$$\begin{aligned}\int_\theta \sum_{i \in N} t_i(v,\theta)\, dG(\theta) &= \sum_{i \in N} \int_{\theta_i} T_i(v,\theta_i) d\theta_i \\ &= \sum_{i \in N} \int_{\theta_i} \left[U_i(v,\theta_i) + \theta_i X_i(v,\theta_i)\right] d\theta_i \\ &= \sum_{i \in N} \left\{\rho_i(v) + \int_{\theta_i} X_i(v,\theta_i) J_i(\theta_i) dG_i(\theta_i)\right\} \\ &= \rho_1(v) + \int_\theta \sum_{i \in N} x_i(v,\theta_i) J_i(\theta_i) dG(\theta), \end{aligned} \tag{4}$$

where $J_i(\theta_i) := \theta_i + \frac{G_i(\theta_i)}{g_i(\theta_i)}$ denotes firm i's virtual cost.

Substituting (4) into the principal's objective function, we can rewrite (LL) as follows:

$$\forall v \in V, \qquad \int_\theta \left\{ \sum_{i\in N} x_i(v,\theta_i)\left[v - J_i(\theta_i)\right] \right\} dG(\theta) \geq \rho_1(v). \tag{LL'}$$

Let $\mu(v) \geq 0$ denote the multiplier associated with this constraint.

The innovating firm's individual rationality simplifies to

$$\forall v \in V, \quad \rho_1(v) \geq 0. \tag{IR'}$$

We let $\nu(v) \geq 0$ denote the multiplier associated with this constraint.

We next focus on the first-order condition for the effort constraint.

$$\int_v \int_\theta \left[\rho_1(v) + \frac{G_1(\theta_1)}{g_1(\theta_1)} x_1(v,\theta)\right] dG(\theta) f_e(v|e) dv \geq c'(e). \tag{MH'}$$

Note that we formulate the condition as a weak inequality to ensure the nonnegativity of the multiplier. Let $\lambda \geq 0$ be the associated multiplier.

Then, $[P]$ can be more succinctly reformulated as follows:

$$\max_{e,x(v,\theta),\rho_1(v)} \quad \int_v \left\{\int_\theta \left[\sum\nolimits_{i\in N} x_i(v,\theta)\left(v - J_i(\theta_i)\right)\right] dG(\theta) - \rho_1(v)\right\} f(v|e) dv$$
$$\text{s.t.} \;\; (LL'), (IR'), (MH').$$

The integrand of the Lagrangian is given by:

$$L(v,\theta,e) := [1+\mu(v)] \left\{ \left[v - \theta_1 - \left(1 - \frac{\beta(v)}{1+\mu(v)}\right) \frac{G_1(\theta_1)}{g_1(\theta_1)} \right] x_1(v,\theta) + \sum_{j\neq 1} \left[v - J_j(\theta_j)\right] x_j(v,\theta) \right\}$$
$$- \rho_1(v)\left[1 + \mu(v) - \nu(v) - \beta(v)\right] - \lambda c'(e),$$

where

$$\beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

The optimal solution $(e, x(v,\theta), \rho_1(v), \lambda, \mu(v), \nu(v))$ must satisfy the following necessary conditions. First, observe that $L(v)$ is linear in $\rho_1(v)$; hence, its coefficient must be equal to zero:

$$1 + \mu(v) - \beta(v) - \nu(v) = 0. \tag{5}$$

The Lagrangian is also linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$x_i(v,\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{\tilde{K}_j(v,\theta_j)\right\} \text{ and } \tilde{K}_i(v,\theta_i) \leq v, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\tilde{K}_i(v,\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta(v)}{1+\mu(v)}\frac{G_i(\theta_i)}{g_i(\theta_i)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1, \end{cases} \quad \text{with } \beta(v) := \lambda \frac{f_e(v|e)}{f(v|e)}.$$

Next, the optimal effort e must satisfy

$$\frac{\partial}{\partial e}\int_v \int_\theta L(v,\theta,e) f(v|e) dv dG(\theta) = 0.$$

Finally, complementary slackness implies that, for each v,

$$\nu(v)\rho_1(v) = 0, \tag{6}$$

$$\mu(v)\left\{\int_\theta \sum_{i\in N} x_i(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta) - \rho_1(v)\right\} = 0, \tag{7}$$

and

$$e\left[\int_v \int_\theta \left[\rho_1(v) + \frac{G_1(\theta)}{g_1(\theta)} x_1(v,\theta)\right] g(\theta) d\theta f_e(v|e) dv - c'(e)\right] = 0. \tag{8}$$

We now provide the characterization. Again there are two cases depending on the value of v. Consider first the case $v < \hat{v}$, where $\beta(v) < 1$. Hence, $1 + \mu(v) - \beta(v) > \mu(v) \geq 0$, and (5) thus implies $\nu(v) > 0$. The complementary slackness condition (6) then yields $\rho_1(v) = 0$. This, together with Lemma 5 (see Appendix D) and the complementary slackness condition (7), implies that $\mu(v) = 0$. Hence, $\tilde{K}_1(v,\theta_1) = J_1(\theta_1) - \beta(v) G_1(\theta_1)/g_1(\theta_1) = K_1(v,\theta_1)$.

Let us now turn to the case $v > \hat{v}$, where $\beta(v) > 1$. Hence, $1 - \beta(v) - \nu(v) < 0$, and (5) thus implies $\mu(v) > 0$; from the complementary slackness condition (7), we thus have

$$\rho_1(v) = \int_\theta \sum_{i\in N} x_i(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta),$$

and Lemma 5 thus implies $\rho_1(v) > 0$ in case $n \geq 2$ or in case $\nu(v) > 0$. The complementary slackness condition (6) then yields $\nu(v) = 0$. It follows now from (5) that $1 + \mu(v) = \beta(v)$. We therefore conclude that $\tilde{K}_1(v,\theta_1) = \theta_1 = K_1(v,\theta_1)$.

The transfer payment t^* follows from (4), with $\rho_j^*(v)$ as described above and $\rho_j^*(v) = 0$ for all $j \neq 1$. The above characterization is valid only when the optimal allocation is monotonic (another necessary condition from incentive compatibility). This follows the assumption that $\frac{G_i(\theta_i)}{g_i(\theta_i)}$ is nondecreasing in θ_i, which implies that $K_i(v,\theta_i) = J_i(\theta_i)$, for $i \neq 1$, and

$$K_1(v,\theta_1) := J_1(\theta_1) - \min\{1,\beta(v)\}\frac{G_1(\theta_1)}{g_1(\theta_1)} = \theta_1 + \max\{0, 1-\beta(v)\}\frac{G_1(\theta_1)}{g_1(\theta_1)},$$

are all nondecreasing in θ_i.

B Proof of Lemma 1

It suffices to note that, starting from any interior effort level e, an increase in effort increases the principal's expected payoff:

$$\frac{\partial}{\partial e}\int_{\underline{v}}^{\bar{v}}\int_{\theta}L(v,\theta,e)dG(\theta)f(v|e)dv=\int_{\underline{v}}^{\bar{v}}\int_{\theta}\sum_{i\in N}x_i(v,\theta)\left(v-J_i(\theta_i)\right)dG(\theta)f_e(v|e)dv>0,$$

where the inequality stems from MLRP, together with the fact that the ex interim expected payoff $\int_\theta \sum_{i\in N} x_i(v,\theta)\left(v-J_i(\theta_i)\right)dG(\theta)$ increases in v.

C Optimality of offering a prize ($\hat{v}<\bar{v}$)

Fix a given environment, namely, a distribution $F(\cdot|e)$ for the value v and, for each firm $i\in N$, a distribution $G_i(\cdot)$ of its cost, and suppose that there exists an optimal mechanism with no monetary reward: $\forall v\in V$, $\rho^*(v)=0$, which amounts to

$$\lambda<\bar{\lambda}:=\frac{f(\bar{v}|e)}{f_e(\bar{v}|e)} \tag{9}$$

and implies $\mu(v)=0$ for any $v\in V$. The optimal allocation is such that $x_i(v,\theta)=0$ for any $v\leq\underline{\theta}$ and, for $v>\underline{\theta}$:

$$x_1^*(v,\theta)=\begin{cases}1 & \text{if } K_1^*(\theta_1)<\min\{v,J_2(\theta_2),...,J_n(\theta_n)\},\\ 0 & \text{otherwise.}\end{cases}$$

The objective of the principal, as a function of e, can thus be expressed as:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i\in N}x_i^*(v,\theta)\left[v-J_i(\theta_i)\right]dG(\theta)\,dF(v|e)\\ + & \lambda\left\{\int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}}X_1^*(v,\theta_1)\,G_1(\theta_1)\,f_e(v|e)d\theta_1dv-c'(e)\right\},\end{aligned}$$

where the innovator's expected probability of obtaining the contract is given by:

$$X_1^*(v,\theta_1)=\int_{\theta_{-1}}x_1^*(v,\theta)\,dG_{-1}(\theta_{-1}).$$

The first-order condition with respect to e yields:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i\in N}x_i^*(v,\theta)\left[v-J_i(\theta_i)\right]dG(\theta)\,f_e(v|e^*)\,dv\\ = & \lambda\left\{c''(e^*)-\int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}}X_1^*(v,\theta_1)\,G_1(\theta_1)\,f_{ee}(v|e^*)d\theta_1dv\right\}.\end{aligned} \tag{10}$$

The optimal effort e^* moreover satisfies the innovator's incentive constraint $c'(e^*) = b(e^*)$, where the innovator's expected benefit is given by:

$$b(e) := \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1) G_1(\theta_1) f_e(v|e) d\theta_1 dv.$$

We now consider several variations of the environment for which the optimal mechanism must involve a prize.

C.1 Reducing cost heterogeneity

Suppose first that costs become less and less heterogeneous: Distributions are now parameterized by m in such a way that, for each firm $i \in N$, its cost becomes distributed according to a distribution $G_i^m(\theta_i)$ over the range $\Theta_i^m = \left[\underline{\theta}, \bar{\theta}^m = \underline{\theta} + \left(\bar{\theta} - \underline{\theta}\right)/m\right]$. For every $m \in \mathbb{N}^*$, we will denote by e^m, λ^m, $K_1^m(\theta_1)$ and $X_1^m(v, \theta_1)$ the values associated with the optimal mechanism.

We first note that, as m goes to infinity, the innovator's effort tends to the lowest level, $\underline{e}$:

Lemma 2. *e^m tends to $\underline{e}$ as m goes to infinity.*

Proof. The innovator's expected benefit becomes

$$b^m(e) := \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v, \theta_1) G_1^m(\theta_1) f_e(v|e) d\theta_1 dv,$$

and satisfies:

$$|b^m(e)| \leq \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\underline{\theta}+\frac{\bar{\theta}-\underline{\theta}}{m}} d\theta_1 \left|f_e(v|e)\right| dv = \frac{\left(\bar{\theta} - \underline{\theta}\right) \int_{\underline{\theta}}^{\bar{v}} \left|f_e(v|e)\right| dv}{m}.$$

Therefore, as m goes to infinity, the expected benefit $b^m(e)$ converges to 0, and the innovator's effort thus converges to the minimal effort, $\underline{e}$. □

Furthermore:

Lemma 3. *As m goes to infinity:*

- *The left-hand side of (10) tends to*

$$B^\infty := \int_{\underline{\theta}}^{\bar{v}} (v - \underline{\theta}) f_e(v|\underline{e}) dv > 0.$$

- *In the right-hand side of (10), the terms within brackets tend to $c''(\underline{e})$.*

PROOF. The left-hand side of (10) is of the form $\int_{\underline{\theta}}^{\bar{v}} h_1^m(v)\,dv$, where

$$h_1^m(v) := f_e(v|e^m)\int_{\underline{\theta}}^{\bar{\theta}^m}\sum_{i\in N} x_i^m(v,\theta)\left[v - J_i(\theta_i)\right]dG^m(\theta).$$

Furthermore, for any $v > \underline{\theta}$, $\hat{J}(\theta) := \min_{i\in N}\{J_i(\theta_i)\} < v$ for m is large enough (namely, for m such that $\bar{\theta}^m < v$, or $m > \left(\bar{\theta} - \underline{\theta}\right)/(v - \underline{\theta})$), and so

$$h_1^m(v) = f_e(v|e^m)\int_{\underline{\theta}}^{\bar{\theta}^m}\left[v - \hat{J}(\theta)\right]dG^m(\theta),$$

which is bounded:

$$|h_1^m(v)| < \left|\max_e f_e(v|e^m)\right| \max\{v - \underline{\theta}, 0\},$$

and converges to

$$\lim_{m\longrightarrow\infty} h_1^m(v) = (v - \underline{\theta})\, f_e(v|\underline{e}).$$

Using Lebesgue's dominated convergence theorem, we then have:

$$\lim_{m\longrightarrow\infty}\int_{\underline{\theta}}^{\bar{v}} h_1^m(v)\,dv = \int_{\underline{\theta}}^{\bar{v}}\lim_{m\longrightarrow\infty} h_1^m(v)\,dv = B^\infty.$$

We now turn to the right-hand side (10). The terms within brackets are

$$c''(e^m) - \int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v,\theta_1)\,G_1^m(\theta_1)\,f_{ee}(v|e^m)d\theta_1 dv,$$

where the first term tends to $c''(\underline{e})$ and the second term is of the form $\int_{\underline{\theta}}^{\bar{v}} h_2^m(v)\,dv$, where

$$h_2^m(v) = f_{ee}(v|e^m)\int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v,\theta_1)\,G_1^m(\theta_1)\,d\theta_1$$

satisfies:

$$|h_2^m(v)| < \max_e |f_{ee}(v|e)|\int_{\underline{\theta}}^{\bar{\theta}^m} d\theta_1 = \frac{\left(\bar{\theta} - \underline{\theta}\right)\max_e |f_{ee}(v|e)|}{m}$$

and thus tends to 0 as m goes to infinity. □

To conclude the argument, suppose that, for any m, the optimal mechanism never involves a prize. Condition (10) should thus hold for any m, and in addition the Lagrangian multiplier

λ^m should satisfy the boundary condition (9). We should thus have:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}^m} & \sum_{i\in N} x_i^m(v,\theta)\left[v-J_i(\theta_i)\right]dG(\theta)\,f_e(v|e^m)\,dv \\ & < \frac{f(\bar{v}|e^m)}{f_e(\bar{v}|e^m)}\left\{c''(e^m)-\int_{\underline{\theta}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}^m} X_1^m(v,\theta_1)\,G_1(\theta_1)\,f_{ee}(v|e^m)d\theta_1 dv\right\}.\end{aligned}$$

Taking the limit as m goes to infinity, this implies:

$$B^\infty = \int_{\underline{\theta}}^{\bar{v}}(v-\underline{\theta})\,f_e(v|\underline{e})\,dv < \frac{f(\bar{v}|\underline{e})}{f_e(\bar{v}|\underline{e})}c''(\underline{e}),$$

which is obviously violated when the return on effort is sufficiently high (e.g., $c''(\underline{e})$ low enough).

C.2 Increasing the number of firms

Let us now keep the cost distributions fixed, and suppose instead that m additional firms are introduced in the environment, with the same cost distribution as the innovator: $G_k(\theta_k) = G_1(\theta_k)$ for $k = n+1, ..., n+m$. Let us again denote by e^m, λ^m, $K_1^m(\theta_1)$ and $X_1^m(v,\theta_1)$ the values associated with the optimal mechanism.

By construction, $K_1^m(\theta_1)\,(>\theta_1) > \underline{\theta}$ for any $\theta_1 > \underline{\theta}$, whereas the lowest $J_j(\theta_j)$ becomes arbitrarily close to $J_1(\underline{\theta}) = \underline{\theta}$ as m increases; it follows that the probability of selecting the innovator, $X_1^m(v,\theta_1)$, tends to 0 as m goes to infinity:

LEMMA 4. *$X_1^m(v,\theta_1)$ tends to 0 as m goes to infinity.*

PROOF. The probability of selecting the innovator satisfies:

$$\begin{aligned} X_1^m(v,\theta_1) &\leq \Pr\left[K_1^m(\theta_1) \leq \min_{j=n+1,\ldots,n+m}\{J_1(\theta_j)\}\right] \\ &\leq \Pr\left[\theta_1 \leq \min_{j=n+1,\ldots,n+m}\{J_1(\theta_j)\}\right] \\ &= \left[1-G_1\left(J_1^{-1}(\theta_1)\right)\right]^m, \end{aligned} \tag{11}$$

where the second inequality stems from $K_1^m(\theta_1) \geq \theta_1$, and the last expression tends to 0 when m goes to infinity. □

It follows that Lemma 2 still holds, that is, the innovator's effort tends to the lowest level, $\underline{e}$ as m goes to infinity. To see this, it suffices to note that the innovator's expected benefit, now equal to

$$b^m(e) = \int_{\underline{\theta}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v, \theta_1) G_1(\theta_1) f_e(v|e) d\theta_1 dv,$$

satisfies:

$$|b^m(e)| \leq \int_{\underline{\theta}}^{\bar{v}} h(v) dv,$$

where

$$h(v) := |f_e(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v, \theta_1) d\theta_1$$

is bounded (by $(\bar{\theta} - \underline{\theta}) \max_{v,e} \{|f_e(v|e)|\}$) and, from the previous Lemma, tends to 0 as m goes to infinity. Hence, as m goes to infinity, the expected benefit $b^m(e)$ converges to 0, and the innovator's effort thus tends to $\underline{e}$.

Likewise, Lemma 3 also holds, that is

- The left-hand side of (10) tends to B^∞. To see this, it suffices to follow the same steps as before, noting that $h_1^m(v)$, now given by

  $$h_1^m(v) = \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^m(v, \theta) [v - J_i(\theta_i)] dG(\theta) f_e(v|e^m),$$

  is still bounded:

  $$|h_1^m(v)| < \max\{v - \underline{\theta}, 0\} \left| \max_e f_e(v|e) \right|,$$

  and tends to $v - \underline{\theta}$ for any $v > \underline{\theta}$:

  - $\hat{J}(\theta) = \min_{i \in N} \{J_i(\theta_i)\}$ is almost always lower than v when m is large enough. Indeed, for any $\varepsilon > 0$, we have:

    $$\begin{aligned} \Pr\left[\hat{J}(\theta) \leq \underline{\theta} + \varepsilon\right] &\geq \Pr\left[\min_{i=n+1,\ldots,n+m} \{J_i(\theta_i)\} \leq \underline{\theta} + \varepsilon\right] \\ &= \Pr\left[\min_{i=n+1,\ldots,n+m} \{\theta_i\} \leq J_1^{-1}(\underline{\theta} + \varepsilon)\right] \\ &= 1 - \left[1 - F\left(J_1^{-1}(\underline{\theta} + \varepsilon)\right)\right]^m, \end{aligned}$$

    where the last expression converges to 1 as m goes to infinity. Therefore, for any $\varepsilon > 0$ there exists $\hat{m}_1(\varepsilon)$ such that, for any $m \geq \hat{m}_1(\varepsilon)$,

    $$\Pr\left[\hat{J}(\theta) \leq \underline{\theta} + \varepsilon\right] \geq 1 - \varepsilon.$$

– Hence, for $m \geq \hat{m}_1(\varepsilon)$:

$$v - \underline{\theta} \geq \int_{\underline{\theta}}^{\bar{\theta}} \sum_{i \in N} x_i^m(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta) \geq (1-\varepsilon)(v - \underline{\theta} - \varepsilon),$$

where the right-hand side converges to $v - \underline{\theta}$ as ε tends to 0.

The conclusion then follows again from Lebesgue's dominated convergence theorem.

- In the right-hand side of (10), the terms within brackets tend to $c''(\underline{e})$. To see this, it suffices to note that $h_2^m(v)$, now given by

$$h_2^m(v) = f_{ee}(v|e^m) \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v,\theta_1) G_1(\theta_1) d\theta_1$$

– is still bounded:

$$\begin{aligned} |h_2^m(v)| &< \max_e |f_{ee}(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v,\theta_1) d\theta_1 \\ &\leq \max_e |f_{ee}(v|e)| \int_{\underline{\theta}}^{\bar{\theta}} \left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m d\theta_1. \end{aligned}$$

– and converges to 0: Indeed, for any $\varepsilon > 0$,

$$\begin{aligned} |h_2^m(v)| &< \max_e |f_{ee}(v|e)| \left\{ \int_{\underline{\theta}}^{\underline{\theta}+\frac{\varepsilon}{2}} d\theta_1 + \int_{\underline{\theta}+\frac{\varepsilon}{2}}^{\bar{\theta}} \left[1 - G_1\left(J_1^{-1}(\underline{\theta}+\varepsilon)\right)\right]^m d\theta_1 \right\} \\ &< \max_e |f_{ee}(v|e)| \left\{ \frac{\varepsilon}{2} + \left(\bar{\theta} - \underline{\theta}\right)\left[1 - G_1\left(J_1^{-1}(\underline{\theta}+\varepsilon)\right)\right]^m \right\}. \end{aligned}$$

But there exists $\hat{m}_2(\varepsilon)$ such that, for any $m \geq \hat{m}_2(\varepsilon)$:

$$\left(\bar{\theta} - \underline{\theta}\right)\left[1 - G_1\left(J_1^{-1}(\theta_1)\right)\right]^m \leq \frac{\varepsilon}{2},$$

and thus

$$|h_2^m(v)| < \max_e |f_{ee}(v|e)| \, \varepsilon.$$

– It follows that the second term converges again to 0:

$$\lim_{m \longrightarrow \infty} \int_{\underline{\theta}}^{\bar{v}} h_2^m(v)\, dv = \int_{\underline{\theta}}^{\bar{v}} \lim_{m \longrightarrow \infty} h_2^m(v)\, dv = 0.$$

The conclusion follows, using the same reasoning as before.

C.3 Increasing the value of the innovation

Let us now keep the supply side (number of firms, and their cost distributions) fixed, and suppose instead that:

- The distribution of v is initially distributed over $V = [\underline{v}, \bar{v}]$; for the sake of exposition we assume $\underline{v} \gg \bar{\theta}$,[16] so that the innovation is always implemented.
- For every $m \in \mathbb{N}^*$, the value v^m becomes distributed over $V^m = [\underline{v}, \bar{v}^m = \underline{v} + m(\bar{v} - \underline{v})]$, according to the c.d.f. $F^m(v^m|e) = F(\underline{v} + (v^m - \underline{v})/m|e)$.

As before, let e^m, λ^m, $K_1^m(\theta_1)$ and $X_1^m(v, \theta_1)$ denote the values associated with the optimal mechanism.

We first note that the virtual costs remain invariant here: $K_i^m(v^m, \theta_i) = K_i(v, \theta_i) = J_i(\theta_i)$ for $i > 1$ and, as

$$\beta^m(v^m) = \lambda \frac{f_e^m(v^m|e)}{f^m(v^m|e)} = \lambda \frac{f_e(v|e)}{f(v|e)},$$

we also have

$$\begin{aligned} K_1^m(v^m, \theta_1) &= J_1(\theta_1) - \min\{\beta^m(v^m), 1\} \frac{G_1(\theta_1)}{g_1(\theta_1)} \\ &= J_1(\theta_1) - \min\{\beta(v), 1\} \frac{G_1(\theta_1)}{g_1(\theta_1)} \\ &= K_1(v, \theta_1). \end{aligned}$$

As by assumption the innovation is always implemented in this variant, the probability of obtaining the contract only depends on these virtual costs, and thus also remains invariant: $x_i^m(v^m, \theta) = x_i^*(v, \theta)$ for any $i \in N$. It follows that, in the right-hand side of (10), the terms within brackets also remained unchanged: Using $X_1^m(v^m, \theta_1) = X_1^*(v, \theta_1)$ and $f_{ee}^m(v^m|e)\,dv^m = f_{ee}(v|e)\,dv$, we have:

$$c''(e) - \int_{\underline{v}}^{\bar{v}^m} \int_{\underline{\theta}}^{\bar{\theta}} X_1^m(v^m, \theta_1) G_1(\theta_1) f_{ee}^m(v^m|e)\, d\theta_1 dv^m = \Gamma^*(e),$$

where

$$\Gamma^*(e) := c''(e) - \int_{\underline{v}}^{\bar{v}} \int_{\underline{\theta}}^{\bar{\theta}} X_1^*(v, \theta_1) G_1(\theta_1) f_{ee}(v|e)\, d\theta_1 dv.$$

[16] Namely, $\underline{v} > \min_{i \in N} \{K_i(\underline{v}, \bar{\theta})\}$.

By contrast, the left-hand side of (10) is unbounded as m goes to infinity: Using $\sum_{i\in N} x_i^*(v,\theta) = 1$ (as by assumption the innovation is always implemented here), $f_e^m(v|e)\,dv^m = f_e(v|e)\,dv$ and $\int_{\underline{v}}^{\bar{v}} f_e(v|e)\,dv = 0$, we have:

$$\begin{aligned}\int_{\underline{v}}^{\bar{v}^m}\int_{\underline{\theta}}^{\bar{\theta}} & \sum_{i\in N} x_i^m(v^m,\theta)\left[v^m - J_i(\theta_i)\right] dG(\theta)\, f_e^m(v^m|e)\,dv^m \\ &= \int_{\underline{v}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}}\sum_{i\in N} x_i^*(v,\theta)\left[\underline{v} + m(v-\underline{v}) - J_i(\theta_i)\right] dG(\theta)\, f_e(v|e)\,dv \\ &= mB^*(e) - C^*(e),\end{aligned}$$

where:

$$\begin{aligned} B^*(e) &= \int_{\underline{v}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}}\sum_{i\in N} x_i^*(v,\theta)\, v dG(\theta)\, f_e(v|e)\,dv = \int_{\underline{v}}^{\bar{v}} v f_e(v|e)\,dv, \\ C^*(e) &= \int_{\underline{v}}^{\bar{v}}\int_{\underline{\theta}}^{\bar{\theta}}\sum_{i\in N} x_i^*(v,\theta)\, J_i(\theta_i)\, dG(\theta)\, f_e(v|e)\,dv. \end{aligned}$$

To conclude the argument, suppose that, for any m, the optimal mechanism never involves a prize. Condition (10) should thus hold for any m, and in addition the Lagrangian multiplier λ^m should satisfy the boundary condition (9). We should thus have:

$$mB^*(e) < C^*(e) + \frac{f(\bar{v}|e)}{f_e(\bar{v}|e)}\Gamma^*(e),$$

which is obviously violated for m large enough.

D Proof of Proposition 3

As earlier, the incentive compatibility constraint can be replaced by the envelope condition:

$$U_i(\mathbf{v},\theta_i) = \rho_i(\mathbf{v}) + \int_{\theta_i}^{\bar{\theta}} X_i(\mathbf{v},\theta_i)d\theta_i, \quad \forall(\mathbf{v},\theta_i)\in V^N\times\Theta, \forall i\in N, \tag{12}$$

where

$$X_i(\mathbf{v},\theta_i) = \mathbb{E}_{\theta_{-i}}\left[\sum_{k\in N} x_i^k(\mathbf{v},\theta_i,\theta_{-i})\right].$$

Using condition (12), we can rewrite the limited liability constraint as:

$$\mathbb{E}_\theta\left[\sum_{k,i\in N} x_i^k(\mathbf{v},\theta)\left\{v^k - J_i(\theta_i)\right\}\right] \geq \sum_{i\in N}\rho_i(\mathbf{v}), \quad \forall\mathbf{v}\in V^n. \tag{LL}$$

Let $\mu(\mathbf{v}) \geq 0$ denote the multiplier associated with this constraint.

Also, from (12) individual rationality boils down to

$$\rho_i(\mathbf{v}) \geq 0, \quad \forall \mathbf{v} \in V^n, \forall i \in N. \tag{IR}$$

We let $\nu_i(\mathbf{v}) \geq 0$ denote the multiplier associated with this constraint.

Finally, using (2) firm i's expected rent and the total expected transfer to the firms can respectively be expressed as

$$\int_{\theta_i} U_i(\mathbf{v}, \theta_i) dG_i(\theta_i) = \rho_i(\mathbf{v}) + \int_{\theta_i} X_i(v, \theta_i) \frac{G_i(\theta_i)}{g_i(\theta_i)} dG_i(\theta_i), \tag{13}$$

and

$$\int_{\theta} \sum_{i \in N} t_i(\mathbf{v}, \theta) dG_i(\theta_i) = \sum_{i \in N} \rho_i(\mathbf{v}) + \int_{\theta} \sum_{i \in N} x_i^k(\mathbf{v}, \theta_i)(J_i(\theta_i) + \psi_i^k) dG_i(\theta_i), \tag{14}$$

and the moral hazard constraint can be replaced by the associated first-order condition, which, using (12), (13), and (14), can be expressed as:[17]

$$\mathbb{E}_{\mathbf{v},\theta}\left[\rho_i(\mathbf{v}) + \frac{G_i(\theta_i)}{g_i(\theta_i)} \sum_k x_i^k(\mathbf{v}, \theta) \middle| e^i, e^{-i}\right] \geq c'(e^i), \quad \forall i \in N. \tag{MH}$$

We formulate again these conditions as weak inequalities to ensure the nonnegativity of the associated multipliers, which we will denote by $\boldsymbol{\lambda} = \left(\lambda^1, ..., \lambda^n\right)$.

The principal's problem can then be more succinctly reformulated as follows:

$$[P] \qquad \max_{\mathbf{x},(\rho_i),\mathbf{e}} \mathbb{E}_{\mathbf{v},\theta}\left[\sum_{k,i \in N} x_i^k(\mathbf{v}, \theta)\left(v^k - J_i(\theta_i) - \psi_i^k\right) - \sum_{i \in N} \rho_i(\mathbf{v}) \middle| \mathbf{e}\right]$$
$$\text{subject to } (LL), (IR), \text{ and } (MH).$$

The analysis of this problem follows the same steps as for the case of a single innovator, and we only sketch them here. The integrand of the Lagrangian is now given by:

$$L(\mathbf{v}, \theta, e) := [1 + \mu(\mathbf{v})] \left\{ \sum_{k,i \in N} \left[v^k - \theta_i - \left(1 - \frac{\beta^i(v^i)}{1 + \mu(\mathbf{v})}\right) \frac{G_i(\theta_i)}{g_i(\theta_i)} - \psi_i^k \right] x_i^k(\mathbf{v}, \theta) \right\}$$
$$- \sum_{i \in N} \rho_i(\mathbf{v}) \left[1 + \mu(\mathbf{v}) - \nu_i(\mathbf{v}) - \beta^i(v^i)\right] - \sum_{i \in N} \lambda^i c'(e^i),$$

where

$$\beta^i(v^i) := \lambda^i \frac{f_e^i(v^i|e)}{f(v^i|e)}.$$

[17] For simplicity, we normalize the firms' efforts in such a way that they face the same cost $c(e)$; any assymmetry can however be accommodated through the distributions $F^k(v^k|e)$.

The first-order condition for the monetary prize $\rho_i(\mathbf{v})$ and for the probability $x_i^k(\mathbf{v},\theta)$ yield, respectively:

$$1+\mu(\mathbf{v})-\nu_i(\mathbf{v})-\beta^i(v^i)=0, \quad \forall \mathbf{v}\in V^n, \forall i\in N, \tag{15}$$

and

$$x_i^k(\mathbf{v},\theta)=\begin{cases} 1 & \text{if } v^k-\tilde{K}_i(\mathbf{v},\theta_i)-\psi_i^k\geq \max\left\{\max_{(l,j)\neq(k,i)} v^l-\tilde{K}_j(\mathbf{v},\theta_j)-\psi_j^l,0\right\}, \\ 0 & \text{otherwise,}\end{cases} \tag{16}$$

where

$$\tilde{K}_i\left(\mathbf{v},\theta_i\right):=J_i(\theta_i)-\frac{\beta^i(v^i)}{1+\mu(\mathbf{v})}\frac{G_i(\theta_i)}{g_i(\theta_i)}.$$

Note that $\tilde{K}_i\left(\mathbf{v},\theta_i\right)$ can be expressed as

$$\theta_i+\left[1-\frac{\beta^i(v^i)}{1+\mu(\mathbf{v})}\right]\frac{G_i(\theta_i)}{g_i(\theta_i)},$$

where (15) and $\nu_i(\mathbf{v})\geq 0$ together imply that the term within brackets is non-negative. It follows that

$$\tilde{K}_i\left(\mathbf{v},\theta_i\right)\geq\theta_i \tag{17}$$

and that $\tilde{K}_i\left(\mathbf{v},\theta_i\right)$ increases with θ_i.

The complementary slackness associated with (LL) implies, for every $\mathbf{v}\in V^n$,

$$\mu(\mathbf{v})\left\{\mathbb{E}_\theta\left[\sum_{k,i\in N}x_i^k(\mathbf{v},\theta)\left\{v^k-J_i(\theta_i)\right\}\right]-\sum_{i\in N}\rho_i(\mathbf{v})\right\}=0, \tag{18}$$

whereas the complementary slackness associated with (IR) implies, for every $i\in N$ and every $\mathbf{v}\in V^n$,

$$\nu_i(\mathbf{v})\rho_i(\mathbf{v})=0. \tag{19}$$

We now prove the following result:

Lemma 5. *Fix any* $\mathbf{v}$ *such that* $\max_{k,i}\left\{v^k-\psi_i^k\right\}>\underline{\theta}$. *We have*

$$\mathbb{E}_\theta\left[\sum_{k,i\in N}x_i^k(\mathbf{v},\theta)\left[v^k-\psi_i^k-J_i(\theta_i)\right]\right]>0, \tag{20}$$

if either (i) $n\geq 2$ *or (ii)* $n=1$ *and either* $v^1-\psi_1^1>\overline{\theta}$ *or* $\nu_1\left(v^1\right)>0$.

PROOF. We first focus on the case in which $n \geq 2$. Fix any $\mathbf{v}$ such that $v^l - \psi_j^l - \underline{\theta} > 0$ for some l, j. And fix any k such that $\sum_i x_i^k(\mathbf{v}, \theta) > 0$ for a positive measure of θ's (a project that does not satisfy this property is never adopted with positive probability and can be ignored).

Consider first the particular case where project k is always implemented and allocated to the same firm i: $x_i^k(\mathbf{v}, .) = 1$ (this can for instance happen when v^k is large and ψ_j^k is prohibitively high for $j \neq i$). In that case:

$$
\begin{aligned}
&\mathbb{E}_\theta \left[\sum_{i \in N} x_i^k(\mathbf{v}, \theta) \left[v^k - \psi_i^k - J_i(\theta_i) \right] \right] \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \left[v^k - \psi_i^k - J_i(\theta_i) \right] dG_i(\theta_i) \\
&= v^k - \psi_i^k - \overline{\theta} \\
&> 0,
\end{aligned}
$$

where the inequality stems from (16), applied to $\theta_i = \overline{\theta}$,[18] and (17).

Let us now turn to the case where no firm is selected with probability 1 to implement project k (because project k is not always implemented, and/or different firms are selected to implement it). By (16), the optimal allocation rule is then such that

$$X_i^k(\mathbf{v}, \theta_i) := \mathbb{E}_{\theta_{-i}} \left[x_i^k(\mathbf{v}, \theta_i, \theta_{-i}) \right]$$

is nonincreasing in θ_i for all $\theta_i \leq v^k - \psi_i^k$ and equals zero for any $\theta_i > v^k - \psi_i^k$. Further, it is strictly decreasing in θ_i for a positive measure of θ_i if $X_i^k(\mathbf{v}, \theta_i) > 0$, and by the choice of k there is at least one such firm.

Now, for every i define

$$\bar{X}_i^k(\mathbf{v}, \theta_i) = \begin{cases} \bar{z}_i^k & \text{if } \theta_i \leq v^k - \psi_i^k \\ 0 & \text{if } \theta_i > v^k - \psi_i^k, \end{cases}$$

where $\bar{z}_i^k$ is a constant in $(0, 1)$ chosen so that

$$\int_{\underline{\theta}}^{\overline{\theta}} \bar{X}_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i) = \bar{z}_i^k G_i(v^k - \psi_i^k) = \int_{\underline{\theta}}^{\overline{\theta}} X_i^k(\mathbf{v}, \theta_i) dG_i(\theta_i).$$

Clearly, $\bar{z}_i^k$, and hence $\bar{X}_i^k(\mathbf{v}, \cdot)$, is well defined.

[18] Generically, this condition implies $v^k - \psi_i^k > \tilde{K}_i\left(\mathbf{v}, \overline{\theta}\right)$; we ignore here the non-generic case $v^k - \psi_i^k = \tilde{K}_i\left(\mathbf{v}, \overline{\theta}\right)$.

We have:

$$
\begin{aligned}
&\mathbb{E}_\theta\left[\sum_{i\in N} x_i^k(\mathbf{v},\theta)\left[v^k-\psi_i^k-J_i(\theta_i)\right]\right]\\
=&\sum_i\int_{\underline{\theta}}^{\overline{\theta}} X_i^k(\mathbf{v},\theta_i)\left[v^k-\psi_i^k-J_i(\theta_i)\right]dG_i(\theta_i)\\
=&\sum_i\int_{\underline{\theta}}^{\min\{\overline{\theta},v^k-\psi_i^k\}} X_i^k(\mathbf{v},\theta_i)\left[v^k-\psi_i^k-J_i(\theta_i)\right]dG_i(\theta_i)\\
>&\sum_i\int_{\underline{\theta}}^{\min\{\overline{\theta},v^k-\psi_i^k\}} \bar{X}_i^k(\mathbf{v},\theta_i)\left[v^k-\psi_i^k-J_i(\theta_i)\right]dG_i(\theta_i)\\
=&\sum_i \bar{z}_i^k\int_{\underline{\theta}}^{\min\{\overline{\theta},v^k-\psi_i^k\}} \left[v^k-\psi_i^k-J_i(\theta_i)\right]dG_i(\theta_i)\\
=&\sum_i \bar{z}_i^k\left(\max\{v^k-\psi_i^k-\overline{\theta},0\}\right)\\
\geq&0.
\end{aligned}
$$

The second equality stems from the fact that $X_i^k(\mathbf{v},\theta_i)=0$ for $\theta_i>v^k-\psi_i^k$ and the strict inequality follows from the fact that $v^k-\psi_i^k-J_i(\theta_i)$ is strictly decreasing in θ_i and, in the relevant range $\left[\underline{\theta},\min\{\overline{\theta},v^k-\psi_i^k\}\right]$, $X_i^k(\mathbf{v},\theta_i)$ is nonincreasing in θ_i, and strictly decreasing in θ_i for a positive measure of θ_i, for some i, whereas by construction $\bar{X}_i^k(\mathbf{v},\cdot)$ is constant and

$$
\int_{\underline{\theta}}^{\min\{\overline{\theta},v^k-\psi_i^k\}} \bar{X}_i^k(\mathbf{v},\theta_i)dG_i(\theta_i)=\int_{\underline{\theta}}^{\min\{\overline{\theta},v^k-\psi_i^k\}} X_i^k(\mathbf{v},\theta_i)dG_i(\theta_i).
$$

Summing the above string of inequalities over all k's, we obtain the desired result.

Next consider the case in which $n=1$. In this case, $X_1^1(\mathbf{v},\theta_1)=x_1^1(\mathbf{v},\theta_1)=1$ for $\tilde{K}_1(v^1,\theta_1)\leq v^1$ and zero otherwise. Since $X_i^k(\mathbf{v},\theta_i)$ is constant when it is strictly positive, the strict inequality above does not follow from the above argument. But the strict inequality does still hold if $v^1-\psi_1^1>\overline{\theta}$ or if $\nu_1\left(v^1\right)>0$.

In the former case, the last inequality above becomes strict, thus yielding the desired result. To consider the latter case, assume without loss $v^1-\psi_1^1\leq\overline{\theta}$. Since $\nu_1\left(v^1\right)>0$, we have $\beta^1(v^1)<1+\mu(v^1)$, so $\tilde{K}_1\left(v^1,\theta_1\right)>\theta_1$, which implies that there exists $\tilde{\theta}<v^1-\psi_1^1$ such that $x_1^1(\mathbf{v},\theta_1)=1$ for $\theta_1<\tilde{\theta}$ and $x_1^1(\mathbf{v},\theta_1)=0$ for $\theta_1>\tilde{\theta}$. Let $\check{\theta}:=\sup\{\theta\leq\overline{\theta}|v^1-\psi_1^1-J_1(\theta)\geq 0\}$. If $\tilde{\theta}\leq\check{\theta}$, then

$$
\mathbb{E}_\theta\left[x_1^1(\mathbf{v},\theta)\left[v^1-\psi_1^1-J_1(\theta_1)\right]\right]=\int_{\underline{\theta}}^{\tilde{\theta}}\left[v^1-\psi_1^1-J_1(\theta_1)\right]dG(\theta_1)>0.
$$

If $\tilde{\theta} < \check{\theta}$, the same result holds since

$$\begin{aligned}
&\mathbb{E}_\theta\left[x_1^1(\mathbf{v},\theta)\left[v^1-\psi_1^1-J_1(\theta_1)\right]\right] \\
=&\int_{\underline{\theta}}^{\tilde{\theta}}\left[v^1-\psi_1^1-J_1(\theta_1)\right]dG(\theta_1) \\
>&\int_{\underline{\theta}}^{\tilde{\theta}}\left[v^1-\psi_1^1-J_1(\theta_1)\right]dG_1(\theta_1)+\int_{\tilde{\theta}}^{v^1-\psi_1^1}\left[v^1-\psi_1^1-J_1(\theta_1)\right]dG_1(\theta_1) \\
=&\int_{\underline{\theta}}^{v^1-\psi_1^1}\left[v^1-\psi_1^1-J_1(\theta_1)\right]dG_1(\theta_1) \\
=&0,
\end{aligned}$$

where the strict inequality holds since $v^1-\psi_1^1-J_1(\theta_1)<0$ for $\theta_1\in(\tilde{\theta},v^1-\psi_1^1)$ (which in turn holds since $\check{\theta}<\tilde{\theta}<v^1-\psi_1^1$), and the last equality follows from integration by parts. □

Without loss of generality, assume $n\geq 2$ (otherwise, there would be a single innovator, a case already studied before). There are two cases. Consider first the case where $\beta^i(v^i)<1$ for every $i\in N$. By (15), we must then have

$$\nu_i(\mathbf{v})=1+\mu(\mathbf{v})-\beta^i(v^i)>0,$$

and the complementary slackness condition (19) thus yields $\rho_i(\mathbf{v})=0$, for every firm $i\in N$. This, together with (20) and the complementary slackness condition (18), implies that $\mu(\mathbf{v})=0$, and thus

$$\tilde{K}_i(\mathbf{v},\theta_1)=J_i(\theta_i)-\beta^i(v^i)\frac{G_i(\theta_i)}{g_i(\theta_i)}:=K_i(v,\theta_1).$$

Consider next the case in which $\max_{i\in N}\left\{\beta^i(v^i)\right\}>1$. Let $\hat{I}=\arg\max_{i\in N}\left\{\beta^i(v^i)\right\}$ the firms that have the highest $\beta^i(v^i)$. Applying (15) to $i\in\hat{I}$ then yields

$$\mu(\mathbf{v})=\nu_i(\mathbf{v})+\beta^i(v^i)-1>\nu_{\hat{\imath}}(\mathbf{v})\geq 0, \tag{21}$$

whereas applying (15) to firm $j\notin\hat{I}$ yields

$$1+\mu(\mathbf{v})-\nu_i(\mathbf{v})=\beta^i(v^i)>\beta^j(v^j)=1+\mu(\mathbf{v})-\nu_j(\mathbf{v}).$$

It follows that $\nu_j(\mathbf{v})>\nu_i(\mathbf{v})\geq 0$, for $i\in\hat{I},j\notin\hat{I}$. Therefore, by complementary slackness (19), $\rho_j(\mathbf{v})=0$, so only firms $i\in\hat{I}$ can receive a positive monetary prize: $\rho_j^*(\mathbf{v})=0$ for $j\notin\hat{I}$. Finally, the complementary slackness condition (18) yields

$$\sum_{i\in\hat{I}}\rho_i^*(\mathbf{v})=\sum_{j\in N}\rho_j^*(\mathbf{v})=\mathbb{E}_\theta\left[\sum_{k,i\in N}x_i^k(\mathbf{v},\theta)\left\{v^k-J_i(\theta_i)\right\}\right].$$

By Lemma 5 the total prize must be strictly positive for all $\mathbf{v}$ such that $v^k > \psi_i^k + \underline{\theta}$ for some k, i. Given the atomlessness of $F_i(\cdot|e)$ for all e, we note that $\hat{I}$ is a singleton almost always, i.e., with probability one. In other words, for any $\mathbf{v}$ such that $v^k > \psi_i^k + \underline{\theta}$ for some k, i, and $\max_i\{\beta^i(v^i)\} > 1$, only one firm receives the monetary prize with probability one.

Last, we derive the characterization of the optimal allocation rule. By the above argument, there exists at least one firm $i \in \hat{I}$ such that $\rho_i^*(\mathbf{v}) > 0$, and for that firm (19) yields $\nu_i(\mathbf{v}) = 0$. But then (15) applied to all $j \in \hat{I}$ along with the fact $\beta^i(v^i) = \beta^j(v^j)$ for $i, j \in \hat{I}$ means that $\nu_i(\mathbf{v}) = 0$ for all $i \in \hat{I}$. It then follows that

$$1 + \mu(\mathbf{v}) = \max_i\{\beta^i(v^i)\}.$$

We thus conclude that

$$\tilde{K}_i(\mathbf{v}, \theta_1) = J_i(\theta_i) - \left(\frac{\beta^i(v^i)}{\max_k \beta^k(v^k)}\right)\left(\frac{G_i(\theta_i)}{g_i(\theta_i)}\right) := K_i(v, \theta_1).$$

Online Appendix
Not for publication

A Forbidding handicaps

Suppose that the innovator cannot be handicapped, compared to its second best allocation. That is, for every v and θ:

$$x_1(v,\theta) \geq x_1^{SB}(v,\theta), \tag{NH}$$

where:

$$x_1^{SB}(v,\theta) := \begin{cases} 1 & \text{if } J_1(\theta_1) \leq \min\{v, \min_{j\neq 1} J_j(\theta_j)\}, \\ 0 & \text{otherwise.} \end{cases}$$

Letting $\alpha(v,\theta) \geq 0$ be the multiplier of the no-handicap constraint (NH), the Lagrangian becomes

$$L(v,e) := [1+\mu(v)]\left\{\left[v - J_1(\theta_1) + \frac{\beta(v)}{1+\mu(v)}\frac{G_1(\theta_1)}{g_1(\theta_1)} + \frac{\alpha(v,\theta)}{1+\mu(v)}\right]x_1(v,\theta) + \sum_{j\neq 1}[v - J_j(\theta_j)]\,x_j(v,\theta)\right\}$$
$$- \rho_1(v)\,[1+\mu(v)-\nu(v)-\beta(v)] - \lambda c'(e) + \alpha(v,\theta)[x_1(v,\theta) - x_1^{SB}(v,\theta)]$$

and the additional complementary slackness is

$$\alpha(v,\theta)\left[x_1(v,\theta) - x_1^{SB}(v,\theta)\right] = 0. \tag{22}$$

The Lagrangian is still linear in x_i's, so the optimal allocation must satisfy, for every i, v, θ:

$$\bar{x}_i(v,\theta) = \begin{cases} 1 & \text{if } i \in \arg\min_j \left\{\bar{K}_j(v,\theta_j)\right\} \text{ and } \bar{K}_i(v,\theta_i) \leq v, \\ 0 & \text{otherwise,} \end{cases}$$

where the shadow cost is now given by:

$$\bar{K}_i(v,\theta_i) := \begin{cases} J_i(\theta_i) - \frac{\beta(v)}{1+\mu(v)}\frac{G_i(\theta_i)}{g_i(\theta_i)} - \frac{\alpha(v,\theta)}{1+\mu(v)} & \text{if } i = 1, \\ J_i(\theta_i) & \text{if } i \neq 1, \end{cases} \quad \text{with } \beta(v) := \lambda\frac{f_e(v|e)}{f(v|e)}.$$

When $v > \tilde{v}$, $\bar{K}_1(v,\theta_1) < J_1(\theta_1)$, and we can thus ignore the constraint (NH); hence $\alpha(v,\theta) = 0$, implying $\bar{K}_1(v,\theta_1) = K_1(v,\theta_1)$ and $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$. Let us now consider the case $v < \tilde{v}$. If $\alpha(v,\theta) = 0$, the above characterization yields again $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$, and $v < \tilde{v}$ then implies $\bar{K}_1(v,\theta_1) > J_1(\theta_1)$ and thus $\bar{x}_1(v,\theta) < x_1^{SB}(v,\theta)$ for at least some θs, contradicting (NH); therefore, we must have $\alpha(v,\theta) > 0$, and the complementary slackness condition (22) thus implies $\bar{x}_1(v,\theta) = x_1^{SB}(v,\theta)$, and thus $\bar{K}_1(v,\theta_1) = J_1(\theta_1)$.

The other constraints are unaffected; thus optimal effort e must satisfy

$$\frac{\partial}{\partial e}\int_v \int_\theta L(v,\theta,e) f(v|e) dv dG(\theta) = 0,$$

and complementary slackness implies that, for each v,

$$\nu(v)\rho_1(v) = 0,$$
$$\mu(v)\left\{\int_\theta \sum_{i\in N} \bar{x}_i(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta) - \rho_1(v)\right\} = 0,$$

and

$$e\left[\int_v \int_\theta \left[\rho_1(v) + \frac{G_1(\theta)}{g_1(\theta)}\bar{x}_1(v,\theta)\right] g(\theta) d\theta f_e(v|e) dv - c'(e)\right] = 0.$$

Going through the same steps as before, and summing-up, we have:

- For $v < \tilde{v}$, $\alpha(v,\theta) = -\beta(v) G_1(\theta_1)/g_1(\theta_1)\,(>0)$ and $\bar{K}_i(v,\theta_i) = J_i(\theta_i)$ for all i (and thus $\bar{x}_i(v,\theta) = x^{SB}(\theta)$ for all i as well).
- For $v > \tilde{v}$, $\alpha(v,\theta) = 0$ and $\bar{x}_i(v,\theta) = x_i^*(v,\theta)$ for all i.

In addition:

- For $v < \hat{v}$, $\nu(v) = 1 - \beta(v) > 0$ and thus $\rho_1(v) = 0$ and $\mu(v) = 0$.
- For $v > \hat{v}$, $\nu(v) = 0$ and $\beta(v) = 1 + \mu(v)$, and thus $\bar{K}_1(v,\theta_1) = \theta_1$ and

$$\rho_1(v) = \int_\theta \sum_{i\in N} \bar{x}_i(v,\theta)\left[v - J_i(\theta_i)\right] dG(\theta).$$

Finally, for $v > \hat{v}$ we have $\bar{x}_1(v,\theta) = x_1^*(v,\theta)$, which is based on $K_1(v,\theta_1) = \theta_1$ and $K_i(v,\theta_i) = J_i(\theta_i)$ for $i \neq 1$; this implies that forbidding handicaps does not affect the size of the monetary prize – even when it affects the multiplier λ.