# ASYMMETRIC PRICE INCREASE IN CRITICAL LOSS ANALYSIS: A REPLY TO DALJORD, SØRGARD, AND THOMASSEN

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## ABSTRACT

When products produced by merging firms or in an antitrust candidate market are differentiated, imposing an asymmetric price increase can be more profitable than raising prices similarly for all products. Critical loss and diversion ratio analyses attempt to model the implications of mergers on competition and antitrust market definition in differentiated markets, and have been a major focus over the last decade by authors such as Michael Katz and Carl Shapiro. James Langenfeld and Wenqing Li, and Øystein Daljord, Lars Sørgard, and Øyvind Thomassen, derive the critical loss formula when the price of only one differentiated product is raised due to a merger. However, they arrive at different results. This article corrects the derivation in Daljord, Sørgard, and Thomassen, and provides the appropriate formula of critical loss with asymmetric price. It then shows how failure to correct Daljord, Sørgard, and Thomassen's formulation of critical loss can lead to incorrect conclusions about market definition and competitive effects in merger analysis, resulting in antitrust markets that tend to be too broad.

## I. INTRODUCTION

James Langenfeld and Wenqing Li,[1] Michael Katz and Carl Shapiro,[2] and Daniel O'Brien and Abraham Wickelgren[3] incorporated diversion ratio into critical loss analysis with differentiated products. Katz and Shapiro, and O'Brien and Wickelgren further derive the relationship between diversion ratio and the critical loss using the Lerner condition. In deriving the relationship

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[1] James Langenfeld & Wenqing Li, *Critical Loss Analysis in Evaluating Mergers*, 46 ANTITRUST BULL. 299 (2001).

[2] Michael L. Katz & Carl Shapiro, *Critical Loss: Let's Tell the Whole Story*, A.B.A. ANTITRUST MAG., Spring 2003, at 49.

[3] Daniel P. O'Brien & Abraham L. Wickelgren, *A Critical Analysis of Critical Loss Analysis*, 71 ANTITRUST L.J. 161 (2003).

between diversion ratio and critical loss, O'Brien and Wickelgren assume symmetry among the products in the candidate antitrust market.[4] Although Katz and Shapiro state that their formulation applies when the price of only one product is increased,[5] Daljord, Sørgard, and Thomassen correctly point out that the result in Katz and Shapiro applies to a symmetric price increase among all products, but not to an asymmetric price increase.[6]

Daljord, Sørgard, and Thomassen attempt to correct Katz and Shapiro's formulation and derive the critical loss formula when the price of only one product is increased, but they also made an error in their formulation. This article corrects Daljord, Sørgard, and Thomassen's formulation of critical loss in the context of asymmetric price increases, and shows how the failure to correct this error can lead to incorrect conclusions about market definition and competitive effects in merger analysis.

In the rest of the article, we first review briefly the formulation of critical loss under the assumption of symmetric price increase, followed by a discussion of why it is important to study asymmetric price increase in merger and acquisition analysis. We then explain the error in Daljord, Sørgard, and Thomassen's formulation and derive the correct critical loss formula with asymmetric price increase. Finally, we show how Daljord, Sørgard, and Thomassen's formulation can lead to incorrect economic inferences.

## II. CRITICAL LOSS WITH SYMMETRIC PRICE INCREASE

Suppose Product 2 and Product 1 are symmetric and the prices of Product 1 and Product 2 are raised by $t$ percent. Denote $m = (p - mc)/p$ as the gross profit margin where $p$ is price and $mc$ is the constant marginal cost. It can be shown that the critical loss associated with a $t$ percent price increase is $t/(t + m)$.[7]

Let $\varepsilon$ denote the absolute value of the own price elasticity. Profit maximization implies that $\varepsilon$ is equal to the inverse of gross profit margin, or $m = 1/\varepsilon$. Using this Lerner condition, Katz and Shapiro, and O'Brien and Wickelgren show that the $t$ percent price increase is profitable if and only if

$$D > \frac{t}{t + m}, \tag{1}$$

where $D$ is the diversion ratio between Product 1 and Product 2, defined as

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[4] *See id.* at 166–67. Symmetry means that the products in the candidate market have the same price, marginal cost, own price elasticity, and the same cross elasticity with each other.

[5] *See* Katz & Shapiro, *supra* note 2, at 53.

[6] *See* Øystein Daljord, Lars Sørgard & Øyvind Thomassen, *The SSNIP Test and Market Definition with the Aggregate Diversion Ratio: A Reply to Katz and Shapiro*, 4 J. COMPETITION L. & ECON. 263 (2008).

[7] *See, e.g.*, Langenfeld & Li, *supra* note 1, at 305.

$\varepsilon_{21}q_2/\varepsilon_{11}q_1$.[8] $\varepsilon_{11}$ is the absolute value of Product 1's own price elasticity and $\varepsilon_{21}$ is the cross elasticity of demand between Product 1 and Product 2.

## III. CRITICAL LOSS WITH ASYMMETRIC PRICE INCREASE

In many cases, the symmetric condition among the products in the merging firm or in the candidate market does not hold. Researchers often find that different brands of consumer products have different own-price elasticities and the cross elasticities between two brands are not equal to each other.[9]

As pointed out by Daljord, Sørgard, and Thomassen, "[t]he U.S. Merger Guidelines leave it an open question whether one should increase one, some, or all prices in the candidate market when performing a SSNIP test" and "[w]hen products in the candidate market are asymmetric, imposing asymmetric SSNIPs may make more economic sense."[10]

### A. Daljord, Sørgard, and Thomassen's Formulation of Critical Loss with Asymmetric Price Increase and Its Correction

Both Langenfeld and Li, and Daljord, Sørgard, and Thomassen derive the critical loss formula when the price of only one product is raised in a differentiated product setting with constant marginal cost. However, they arrive at different results.

Langenfeld and Li use inequality (2) below,

$$
\begin{aligned}
&(p_1 + \Delta p_1 - mc_1)(q_1 - \Delta q_1) + (p_2 - mc_2)(q_2 + \Delta q_1 D_{12}) \\
&\geq (p_1 - mc_1)q_1 + (p_2 - mc_2)q_2
\end{aligned}, \tag{2}
$$

where $D_{12}$ is the diversion ratio from Product 1 to Product 2, to derive critical loss ("$CL$") as:[11]

$$
CL_1 = \frac{t_1}{m_1 + t_1 - m_2 \dfrac{p_2}{p_1} D_{12}}, \tag{3}
$$

where $t_1$ is the percent of price increase in $p_1$ and $D_{12} = \dfrac{\varepsilon_{21}q_2}{\varepsilon_{11}q_1}$.

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[8]  *See* Katz & Shapiro, *supra* note 2, at 56; O'Brien & Wickelgren, *supra* note 3, at 171.

[9]  *See, e.g.*, Jerry Hausman, Gregory Leonard & J. Douglas Zona, *Competitive Analysis with Differentiated Products*, 34 ANNALS OF ECON. & STAT. 159 (1994) (discussing price elasticity estimates on beer); Inseong Song & Pradeep K. Chintagunta, *Measuring Cross-Category Price Effects with Aggregate Store Data*, 52 MGMT. SCI. 1594 (2006) (discussing price elasticity estimates on liquid detergent, powered detergent, liquid softener, and sheet softener); James Langenfeld, Wenqing Li & Sophie Yang, *Bayer or Walgreens', Scott or Dominick's? Competition Between National and Store Brands* (Working Paper) (on file with authors) (discussing price elasticity estimates on bathroom tissue products). *See also* Daljord, Sørgard & Thomassen, *supra* note 6 (discussing the issue of asymmetry).

[10]  *See* Daljord, Sørgard & Thomassen, *supra* note 6, at 270.

[11]  *See* Langenfeld & Li, *supra* note 1, at 336–37.

If the actual loss ("*AL*") is smaller than *CL*, or

$$CL_1 = \frac{t_1}{m_1 + t_1 - m_2 \frac{p_2}{p_1} D_{12}} \geq AL = \frac{\Delta q_1}{q_1}, \tag{4}$$

then the price increase of $t_1$ percent for Product 1 is profitable.

Daljord, Sørgard, and Thomassen use the following equation to derive the critical loss:

$$((1 + t_1)p_1 - mc_1)q_1(1 - CL) - (p_1 - mc_1)q_1 \\ + (p_2 - mc_2)q_2(1 + t \cdot \varepsilon_{21}) - (p_2 - mc_2)q_2 = 0, \tag{5}$$

where $\varepsilon_{21}$ is the cross elasticity of demand for Product 2 with respect to the price of Product 1.[12]

Rearrange some of the terms in equation (5), and it can be shown that equation (5) is equivalent to equation (6) below:

$$((1 + t_1)p_1 - mc_1)(q_1 - q_1 CL) - (p_1 - mc_1)q_1 \\ + (p_2 - mc_2)(q_2 + t \cdot \varepsilon_{11} \cdot q_1 \cdot D_{12}) - (p_2 - mc_2)q_2 = 0, \tag{6}$$

and again $D_{12} = \frac{\varepsilon_{21} q_2}{\varepsilon_{11} q_1}$.

The critical loss derived by Daljord, Sørgard, and Thomassen is:

$$CL_2 = \frac{t_1}{m_1 + t_1}(1 + \lambda D_{12}), \tag{7}$$

where $\lambda = (p_2 - mc_2)/(p_1 - mc_1)$.[13]

The reason the critical loss in Daljord, Sørgard, and Thomassen is different from that derived by Langenfeld and Li is because there is an internal inconsistency in equations (5) and (6) above which were used by Daljord, Sørgard, and Thomassen. More specifically, the term $(q_1 CL)$, in equation (6) is the unit sales loss for Product 1 associated with the *critical loss*. However, the term $(t \cdot \varepsilon_{11} \cdot q_1 \cdot D_{12})$ in equation (6) represents the sales diversion to Product 2 based on the *actual sales loss* of Product 1 as the price of Product 1 increases by $t$ percent.

In contrast, the inequality (2) used by Langenfeld and Li is internally consistent. If one interprets $\Delta q_1$ in inequality (2) as the actual sales loss for Product 1 as its price is raised by the amount of $\Delta p_1$, then $\Delta q_1 D_{12}$ in inequality (2) represents the sales diversion to Product 2 based on the actual sales loss for Product 1. On the other hand, if one interprets $\Delta q_1$ in inequality (2) as critical sales loss for Product 1 that makes inequality (2) become an equality, then

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[12] *See* Daljord, Sørgard & Thomassen, *supra* note 6, at 268.
[13] *Id.* at 269.

$\Delta q_1 \, D_{12}$ in inequality (2) represents the sales diversion to Product 2 based on the critical sales loss for Product 1.

We can correct the internal inconsistency in Daljord, Sørgard, and Thomassen's derivation by using the sales diversion to Product 2 based on the critical sales loss for Product 1. If we make this correction, equation (6) becomes:

$$((1 + t_1)p_1 - mc_1)(q_1 - q_1 CL) - (p_1 - mc_1)q_1 \\ + (p_2 - mc_2)(q_2 + CL \cdot q_1 \cdot D_{12}) - (p_2 - mc_2)q_2 = 0. \tag{8}$$

Rearrange the terms, equation (8) becomes:

$$t_1 p_1 q_1 - (p_1 - mc_1 + t_1 p_1)(q_1 CL) + (p_2 - mc_2)(CL \cdot q_1 \cdot D_{12}) = 0. \tag{9}$$

Using (9) to solve for $CL$, it can be shown that

$$CL = \frac{t_1}{m_1 + t_1 - m_2 \dfrac{p_2}{p_1} D_{12}} = CL_1. \tag{10}$$

In other words, if one corrects the error in Daljord, Sørgard, and Thomassen's derivation, we arrive at the same critical loss formula as derived by Langenfeld and Li.

## B. The Critical Loss Expression with Asymmetric Price Increase in Langenfeld and Li is Confirmed by Moresi, Salop, and Woodbury

In this part, we show that critical loss formula for asymmetric price increase derived by Langenfeld and Li is consistent with the results in Serge Moresi, Steven Salop, and John Woodbury. Moresi, Salop, and Woodbury extend the analysis in Katz and Shapiro, and in O'Brien and Wickelgren by considering multi-product firms.[14] Moresi, Salop, and Woodbury also derive the general condition for profit-maximizing price increase if the price of only one product is increased when firms produce multiple products.

Using the notation in Moresi, Salop, and Woodbury, let $M$ represent both the number and the set of products in the candidate market. For each Product $j \in M$, let $N_j$ be the set of all products sold by the owner of Product $j$.[15] As stated in Moresi, Salop, and Woodbury, "under the assumption that demands and costs are linear, if a price increase of $Z$ is neither profitable nor unprofitable, then the profit maximizing price increase is $Z/2$."[16]

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[14] *See* Serge X. Moresi, Steven C. Salop & John R. Woodbury, *Implementing the Hypothetical Monopolist SSNIP Test with Multi-Product Firms*, ANTITRUST SOURCE 1 (Feb. 2008), http://www.americanbar.org/content/dam/aba/publishing/antitrust_source/Feb08_Moresi.authcheckdam.pdf.

[15] *Id.* at 6.

[16] *Id.*

If the hypothetical monopolist would raise the price of Product $j$ only, Moresi, Salop, and Woodbury show that the profit-maximizing price increase would be equal to:[17]

$$t_j^* = \frac{1}{2}\left[\sum_{k\neq j}^{M} D_{jk} m_k \frac{p_k}{p_j} - \sum_{k\neq j}^{N_j} D_{jk} m_k \frac{p_k}{p_j}\right]. \tag{11}$$

Consider a special case where there are only two firms and each firm produces one product only, and the hypothetical monopolist would raise the price of Product 1 only. In this case, the second term inside the bracket in (11) disappears because each firm only produces one product, and the first term inside the bracket in (11) becomes $m_2 p_2 / p_1 D_{12}$. Therefore, the profit-maximizing price increase in Moresi, Salop, and Woodbury is simplified to:

$$t_1^* = \frac{1}{2} m_2 \frac{p_2}{p_1} D_{12}. \tag{12}$$

The Lerner condition implies that $\Delta q_1 / q_1 = t_1 / m_1$. Substitute this expression into Langenfeld and Li's formulation of critical loss with asymmetric price increase, inequality (4) above, then inequality (4) holds, if and only if:

$$m_2 \frac{p_2}{p_1} D_{12} \geq t_1. \tag{13}$$

Inequality (13) implies that the "break-even" price increase is equal to $m_2 p_2 / p_1 D_{12}$, and thus assuming demand and cost are linear, the profit-maximizing price increase is also equal to $1/2 m_2 p_2 / p_1 D_{12}$ under Langenfeld and Li's formulation, which is the same as that in Moresi, Salop, and Woodbury's formulation.

## C.  Daljord, Sørgard, and Thomassen's Critical Loss Formula can lead to Incorrect Conclusions in Market Definition and Competitive Effects Analysis

To illustrate how the error in Daljord, Sørgard, and Thomassen's critical loss formulation can lead to incorrect conclusions, we define

$$k = \frac{m_2 p_2}{m_1 p_1} D_{12}. \tag{14}$$

It can be shown the difference between Langenfeld and Li's critical loss formula, $CL_1$, and Daljord, Sørgard, and Thomassen's critical loss formula,

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[17] *Id.* at 7, equation (8). $t_j^*$ and $D_{jk}$ in *supra* equation (7) is equivalent to $X_j^*$ and $\delta_{jk}$ in equation (8) in *id.*

$CL_2$, is equal to:

$$CL_1 - CL_2 = \frac{kt_1(km_1 - t_1)}{(m_1 + t_1 - m_1 k)(m_1 + t_1)}. \qquad (15)$$

As can be seen from equations (14) and (15), the difference between the correct critical loss measure derived by Langenfeld and Li and the critical loss measure derived by Daljord, Sørgard, and Thomassen is bigger, the bigger the price ratio of Product 2 to Product 1, the bigger the diversion ratio Product 1 to Product 2, the bigger the gross margin of Product 2, and the smaller the gross margin of Product 1.

In particular, because $CL_1$ should be greater than zero, $(m_1 + t_1 - m_1 k)$ in the denominator of (15) is greater than zero. Therefore,

$$\text{sign of}(CL_1 - CL_2) = \text{sign of}(km_1 - t_1) = \text{sign of} \frac{m_2 p_2}{p_1} D_{12} - t_1. \qquad (16)$$

To further assess how the critical loss measure derived by Daljord, Sørgard, and Thomassen diverge from the correct critical loss measure, we conduct a Monte Carlo style simple simulation (10,000 draws) of the values of $CL_1$ and $CL_2$ that result from equation (15) under the assumption of 5 percent price increase for $t_1$ and uniform probability distribution for $m_1$, $m_2$, $D_{12}$, and $p_2/p_1$. Some researchers have stated that price-cost margins are often in the range of 40 to 70 percent,[18] so $m_1$ and $m_2$ are assumed to range from 0.4 to 0.7. $D_{12}$ is assumed to range from 0.1 to 0.5. If Product 1 and Product 2 are substitutes, their price ratio is unlikely to be very big and $p_2/p_1$ is assumed to range from 0.75 to 1.25.

Given the range of $m_1$, $m_2$, $D_{12}$, and $p_2/p_1$ assumed above, the critical loss measured derived by Daljord, Sørgard, and Thomassen is almost always smaller than the correct critical loss measure derived by Langenfeld and Li. Among the 10,000 draws, there are only 166 instances where $(CL_1 - CL_2) < 0$, and in these instances the average value of $(CL_1 - CL_2)$ is only −0.000070433.

Figures 1, 2, and 3 below show the average values of $CL_1$ and $CL_2$ from different assumed ranges for $D_{12}$, $p_2/p_1$, and $m_2/m_1$, and assumed price increase for Product 1 of 5 percent. As can be seen from Figures 1, 2, and 3, average values of $CL_1$ are always greater than $CL_2$ and the magnitude of $(CL_1 - CL_2)$ becomes larger as $D_{12}$, $p_2/p_1$, and $m_2/m_1$ increase. Figures 1, 2, and 3 also show that the difference between $CL_1$ and $CL_2$ becomes larger as $CL_1$ increases.

For example, as shown in Figure 1, when the diversion ratio from Product 1 to Product 2 is between 0.10 and 0.15, the correct critical loss is about 9.7

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[18] Gregory J. Werden, for example, has stated that "price-cost margins in real-world antitrust matters commonly are in the 40 to 70 percent range." Gregory J. Werden, *Demand Elasticities in Antitrust Analysis*, 66 ANTITRUST L.J. 363, 390 (1998).
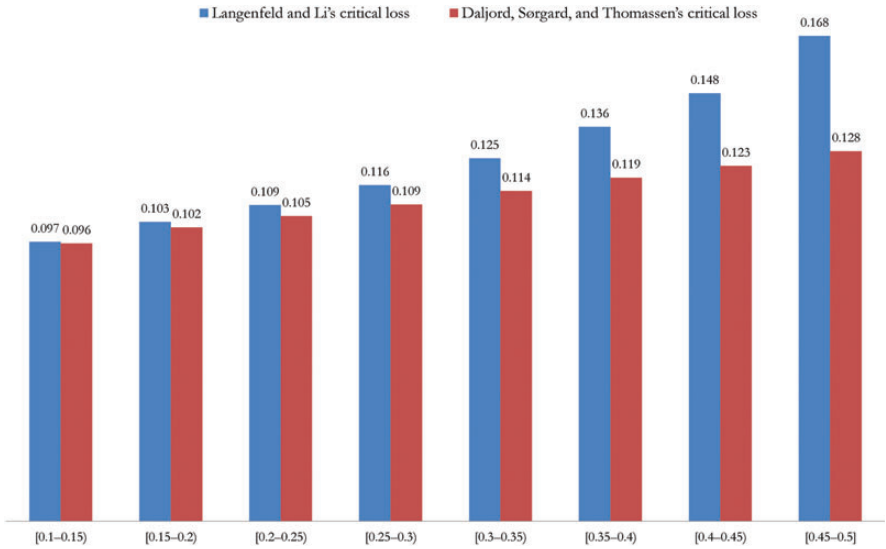
**Figure 1.** Daljord, Sørgard, and Thomassen's critical loss versus Langenfeld and Li's critical loss, by range of diversion ratio from Product 1 to Product 2
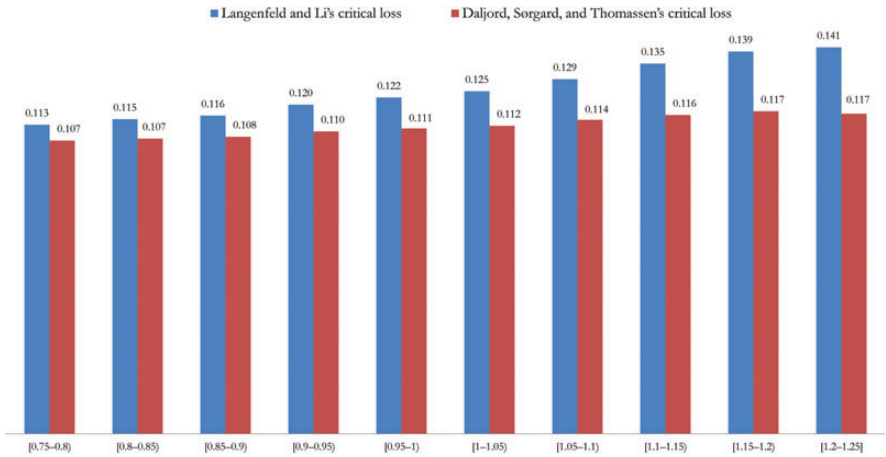


**Figure 2.** Daljord, Sørgard, and Thomassen's critical loss versus Langenfeld and Li's critical loss, by range of price ratio from Product 2 and Product 1

percent on average, while the critical loss derived by Daljord, Sørgard, and Thomassen is about the same at 9.6 percent on average. As the range of the diversion ratio from Product 1 to Product 2 increases to between 0.45 and 0.50, the correct critical loss is about 17 percent on average, while the critical loss derived by Daljord, Sørgard, and Thomassen is only about 13 percent on average. As a result, when the diversion ration from Product 1 to Product 2 is
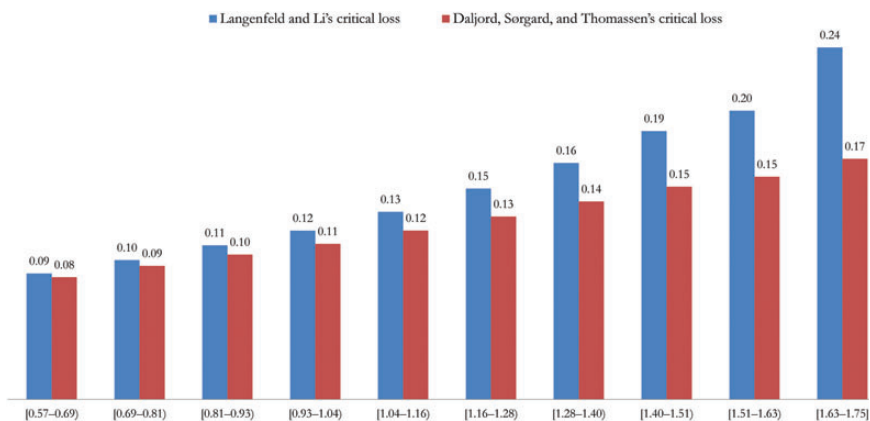
■ Langenfeld and Li's critical loss     ■ Daljord, Sørgard, and Thomassen's critical loss



**Figure 3.** Daljord, Sørgard, and Thomassen's critical loss versus Langenfeld and Li's critical loss, by range of gross profit margin ratio between Product 2 and Product 1

high, it is more likely that the Daljord, Sørgard, and Thomassen's critical loss will lead to incorrect conclusions.

For example, suppose the diversion ratio from Product 1 to Product 2 is between 0.45 and 0.5, and the absolute value of the demand elasticity for Product 1 is 3. Then given a 5 percent price increase, the actual loss of sales for Product 1 is 15 percent. Using Daljord, Sørgard, and Thomassen's critical loss measure, it is likely that one will wrongly conclude that the price increase is not profitable because critical loss is only 13 percent on average, and thus, that there is no anticompetitive effect, or that the antitrust market is broader than the current candidate market. However, if one uses the correct critical loss measure, one will reach the correct conclusion that the price increase is likely profitable because the critical loss is 17 percent on average. Therefore, there are potential anticompetitive effects, or the candidate market can constitute a relevant market.

To summarize, simulation analysis indicates that the critical loss measure derived by Daljord, Sørgard, and Thomassen tends to be smaller than the correct critical loss measure. Consequently, the critical loss measure derived by Daljord, Sørgard, and Thomassen tends to define an antitrust market that is too broad or to overlook potential anticompetitive effects when they exist. This is especially true when the diversion ration from Product 1 to Product 2 is high, and when the price ratio and gross profit margin ratio between Product 2 and Product 1 is high.